EE3123 Introduction to Electric Power Systems

Review of AC Power Circuits and Components

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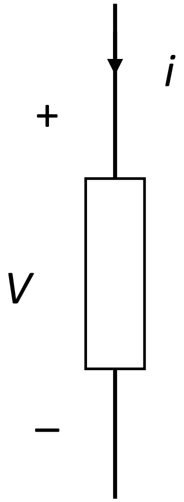
Many thanks to Prof. Michael Tse

Voltage and Current

Trigonometric functions



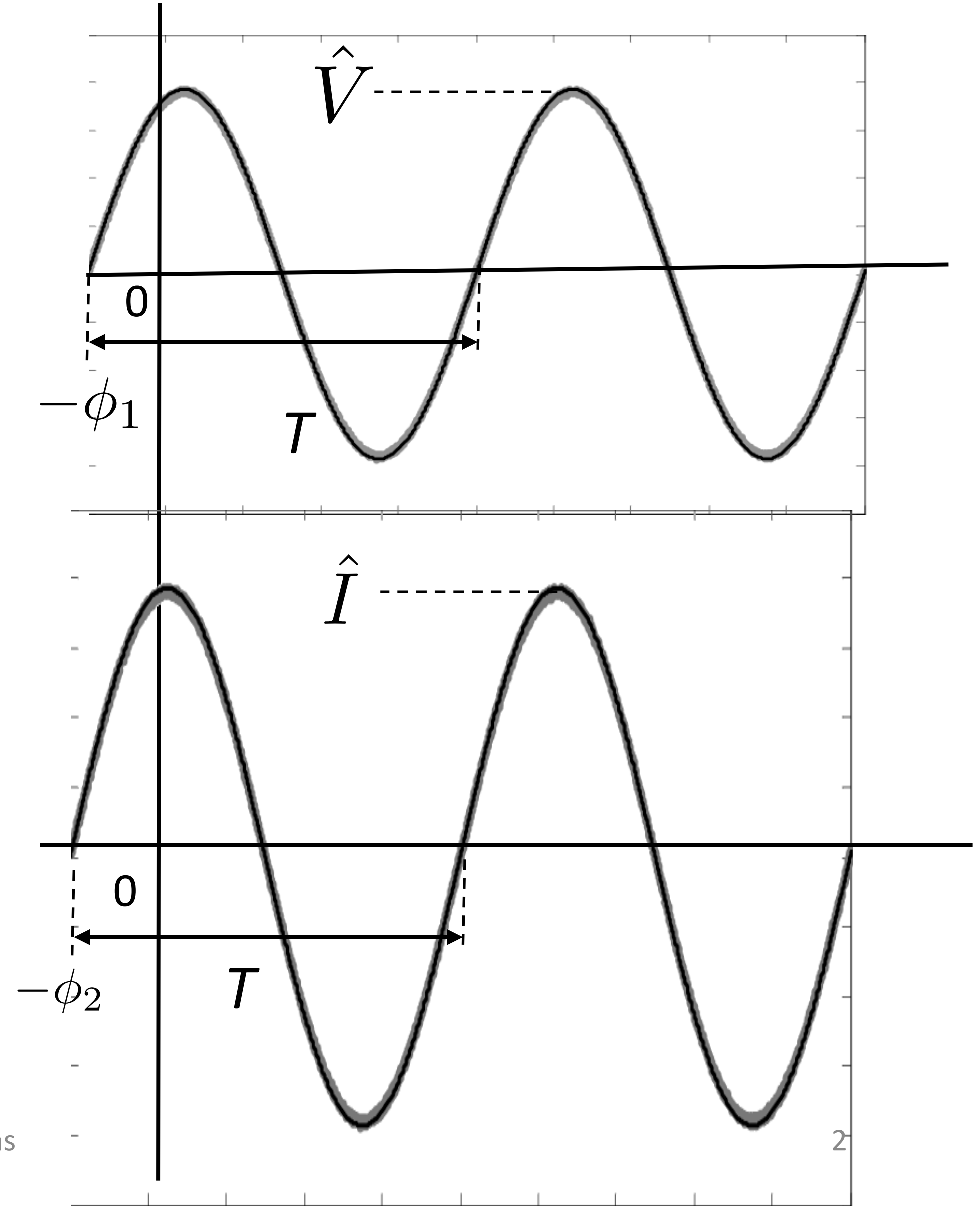
$$v = \hat{V}\sin(\omega t + \phi_1)$$

$$i = \hat{I}\sin(\omega t + \phi_2)$$

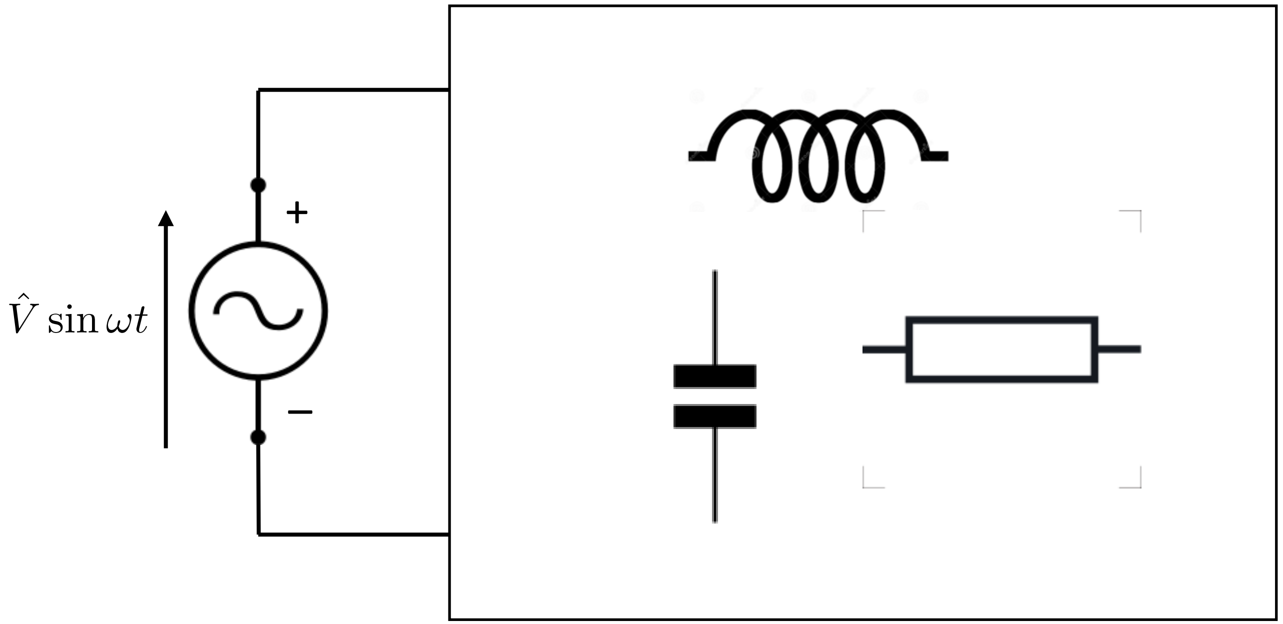
$$\omega = 2\pi f = \frac{2\pi}{T}$$

rad/s

cycle/s or
hertz or Hz



AC circuits driven by a sinusoidal source



For a linear circuit driven by a sinusoidal source, all voltages and currents in the circuit will be sinusoidal and at the SAME frequency.

Each voltage or current may have different magnitude and phase angle, but SAME frequency.

Representing sine waves of SAME frequency

A way to represent many sine functions of same frequency.

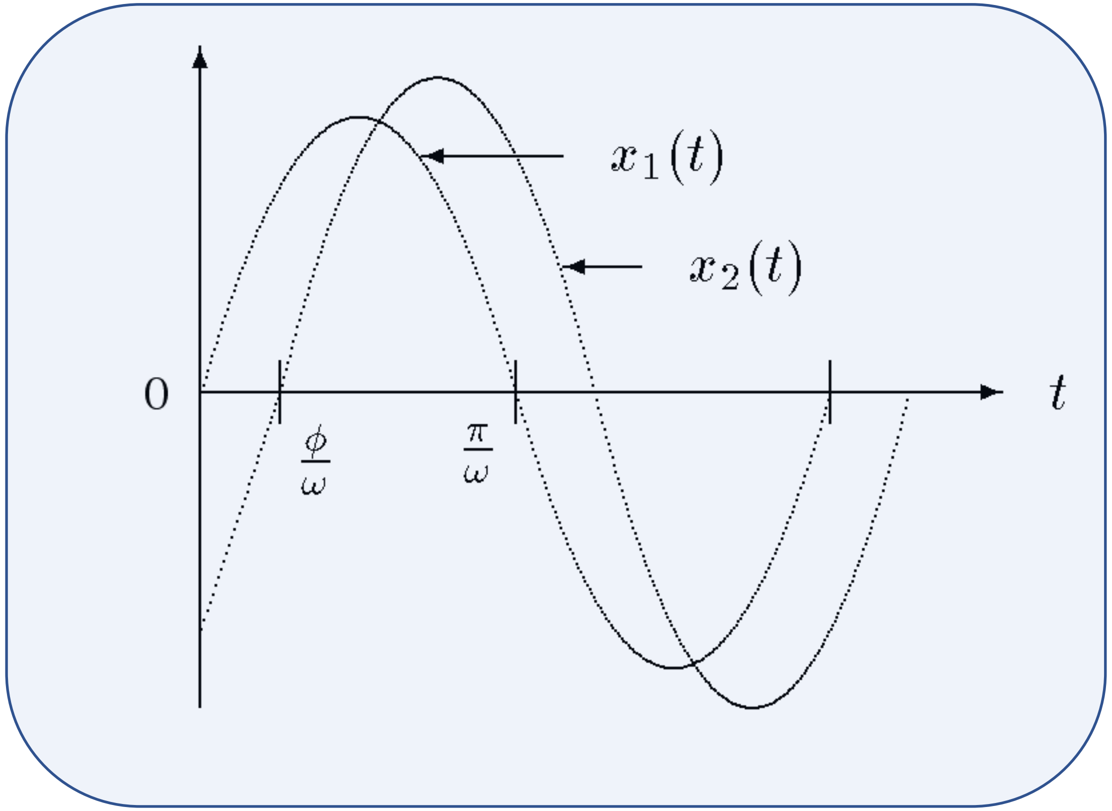
Consider 2 sine functions:

$$x_1(t) = X_1 \sin \omega t$$

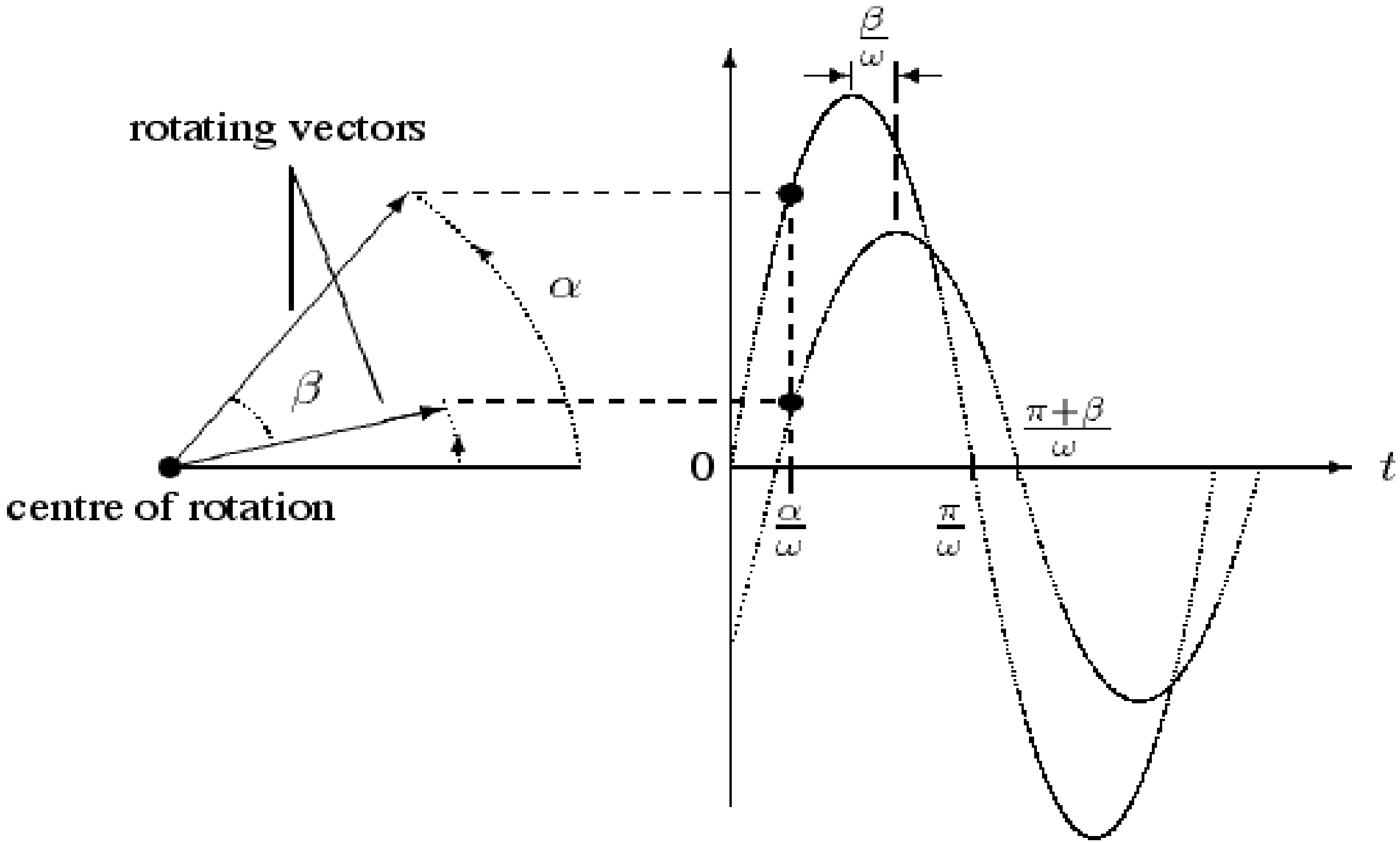
$$x_2(t) = X_2 \sin(\omega t - \phi)$$

Two variables:

- Amplitude (max value)
- and phase angle.



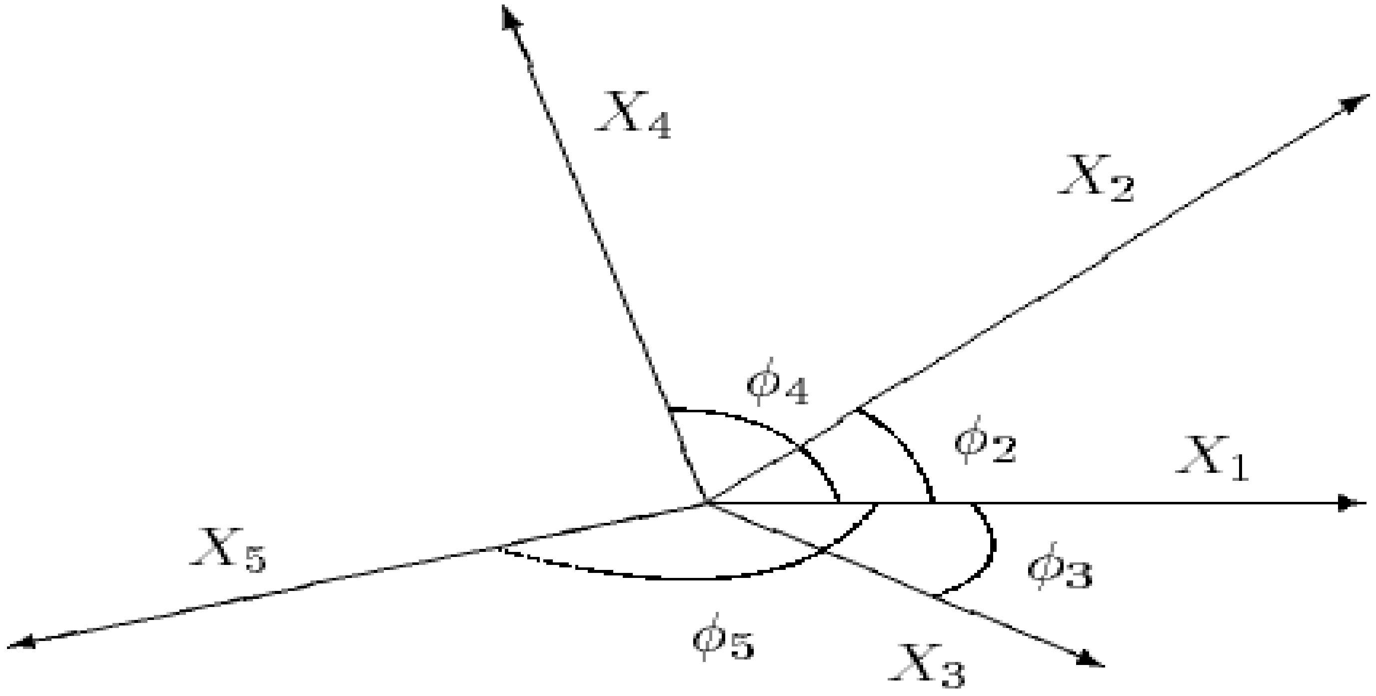
Rotating vector: PHASOR



Phasor diagram

- Consider 5 sine functions of same frequency:

$$
\begin{aligned}
x_1(t) &= X_1 \sin \omega t && \textbf{reference phase} \\
x_2(t) &= X_2 \sin(\omega t + \phi_2) && \textbf{leading } x_1 \textbf{ by } \phi_2 \\
x_3(t) &= X_3 \sin(\omega t - \phi_3) && \textbf{lagging } x_1 \textbf{ by } \phi_3 \\
x_4(t) &= X_4 \sin(\omega t + \phi_4) && \textbf{leading } x_1 \textbf{ by } \phi_4 \\
x_5(t) &= X_5 \sin(\omega t - \phi_5) && \textbf{lagging } x_1 \textbf{ by } \phi_5
\end{aligned}
$$



Phasors

A sinusoidal voltage or current at constant frequency

$$v(t) = \mathrm{V}_{\max} \cos(\omega t + \delta)$$

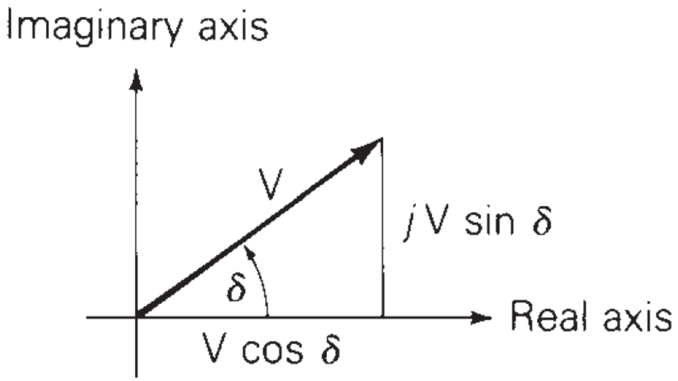
The root-mean-square (rms) value, also called effective value, of the sinusoidal voltage is

$$\mathrm{V} = \frac{\mathrm{V}_{\max}}{\sqrt{2}}$$

The rms phasor representation of the voltage is given in two forms

—polar and rectangular

$$V = \underbrace{\mathrm{V}\angle\delta}_{\text{polar}} = \underbrace{\mathrm{V}\cos\delta + j\mathrm{V}\sin\delta}_{\text{rectangular}}$$



Phasor diagram for converting from polar to rectangular form

Phasors - Example

A phasor can be easily converted from one form to another.

- As an example, the voltage

$$v(t) = 169.7 \cos(\omega t + 60^\circ) \quad \text{volts}$$

has a maximum value V_{max} =169.7 volts, a phase angle δ = 60° when referenced to cos(ωt), and an rms phasor representation in polar form of

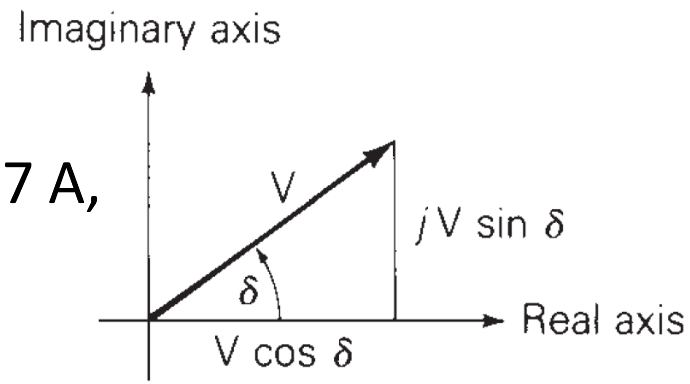
$$V = 120\underline{/60^\circ} \quad \text{volts}$$

- Also, the current

$$i(t) = 100 \cos(\omega t + 45^\circ) \quad \text{A}$$

has a maximum value I_{max} = 100 A, an rms value I = 100/(√2)=70.7 A, a phase angle of 45 °, and a phasor representation

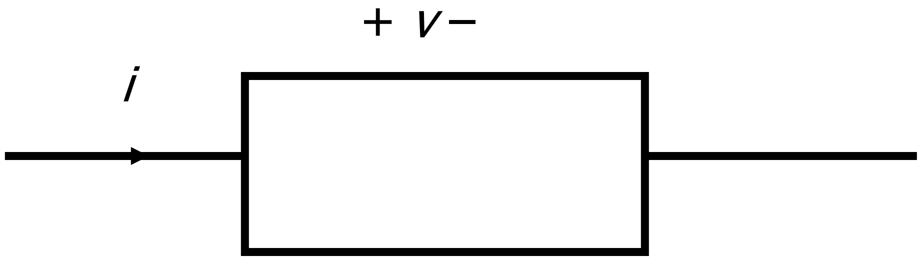
$$I = 70.7\underline{/45^\circ} = 50 + j50 \quad \text{A}$$



Phasor diagram for converting from polar to rectangular form

Basic Relations

Instantaneous power in single-phase ac circuits



Component constitutive relations

- Resistor

$$v_R = i_R R \quad \longrightarrow \text{If } v_R = V_R \sin \omega t, \text{then } i_R = (v_R/R) \sin \omega t.$$

- Inductor

$$v_L = L\frac{di_L}{dt} \quad \longrightarrow \text{If } i_L = I_L \sin \omega t, \text{then } v_L = I_L \omega L \cos \omega t$$

(derivative of current)

- Capacitor

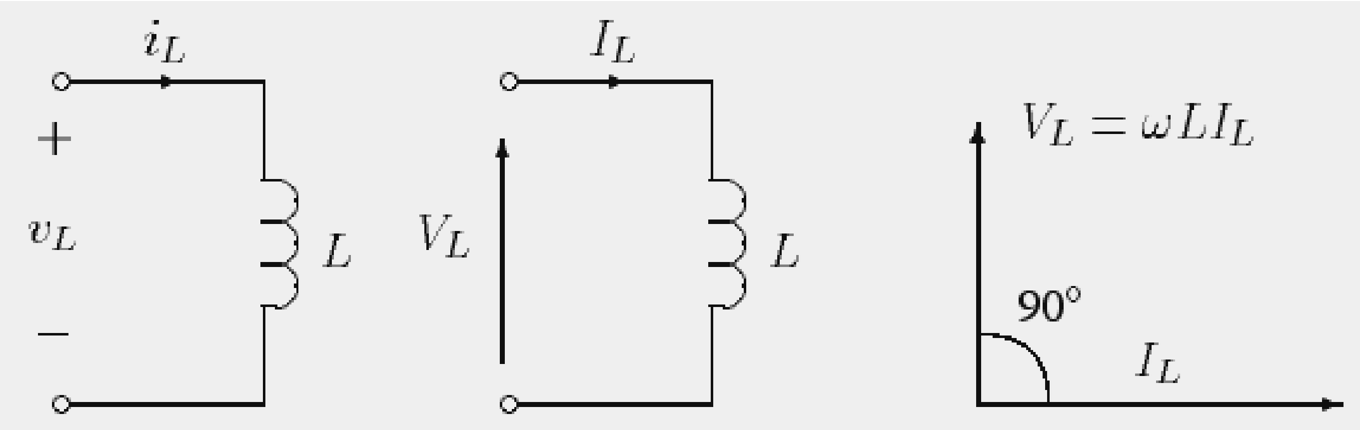
$$i_C = C\frac{dv_C}{dt} \quad \longrightarrow \text{If } v_C = V_C \sin \omega t, \text{then } i_C = V_C \omega C \cos \omega t.$$

(derivative of voltage)

Phasor relations

INDUCTOR $v_L = L\frac{di_L}{dt}$

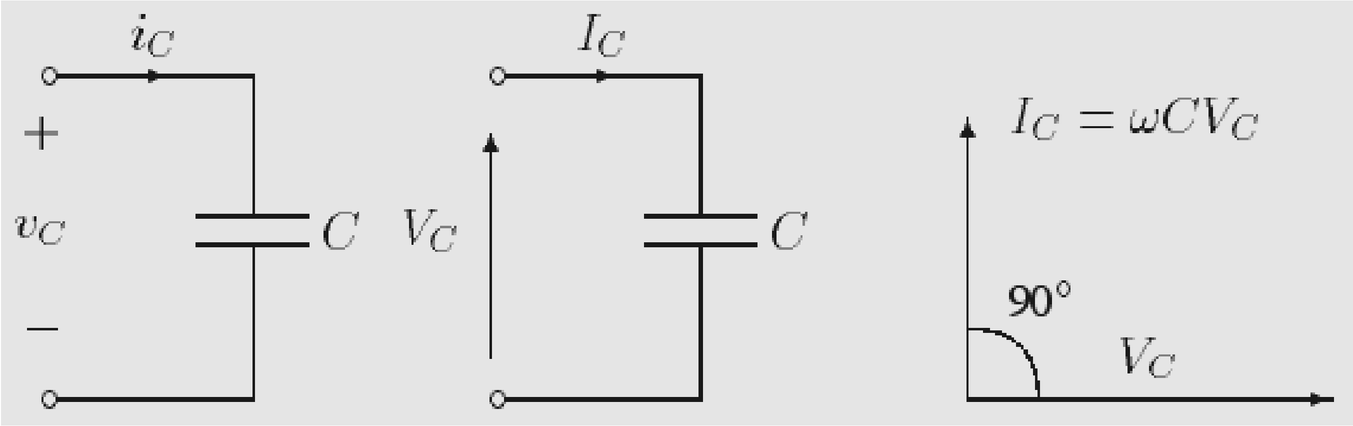
If $i_L = I_L \sin \omega t$, then $v_L = \omega L\, I_L \cos \omega t$.
or $v_L = \omega L\, I_L \sin(\omega t + 90^o)$



Phasor relations

CAPACITOR $i_C = C\frac{dv_C}{dt}$

If $v_C = V_C \sin \omega t$, then $i_C = \omega C\, V_C \cos \omega t$.
or $i_C = \omega C\, V_C \sin(\omega t + 90^o)$



Amplitude and phase

Inductor

Voltage-to-current ratio = ωL

Voltage *leads* current by *90°*

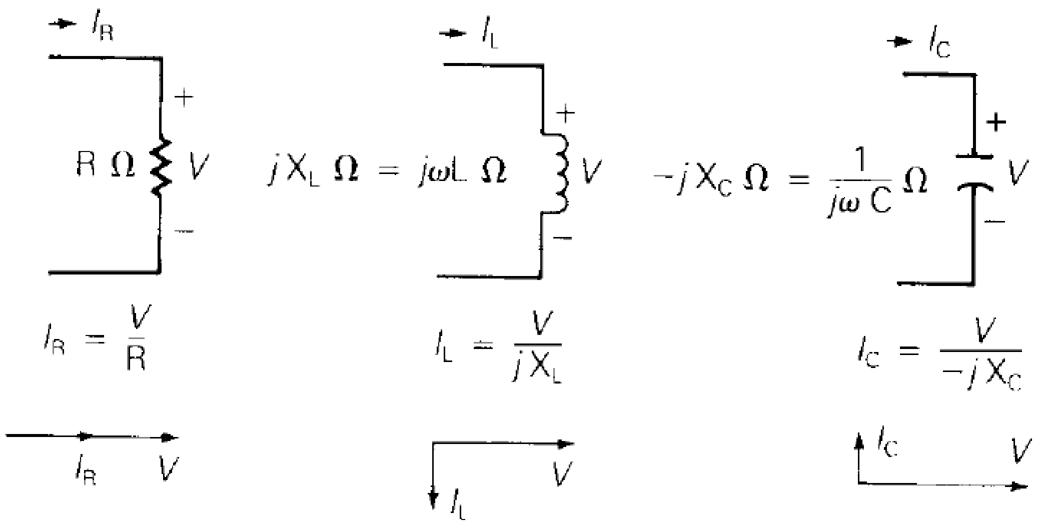
Capacitor

Voltage-to-current ratio = $1 / \omega C$

Voltage *lags* current by *90°*

$$\left.\begin{array}{c} \omega L \\ \\ \dfrac{1}{\omega C} \end{array}\right\}$$

are like resistance, but we call them "**reactance**"; unit is still Ω.

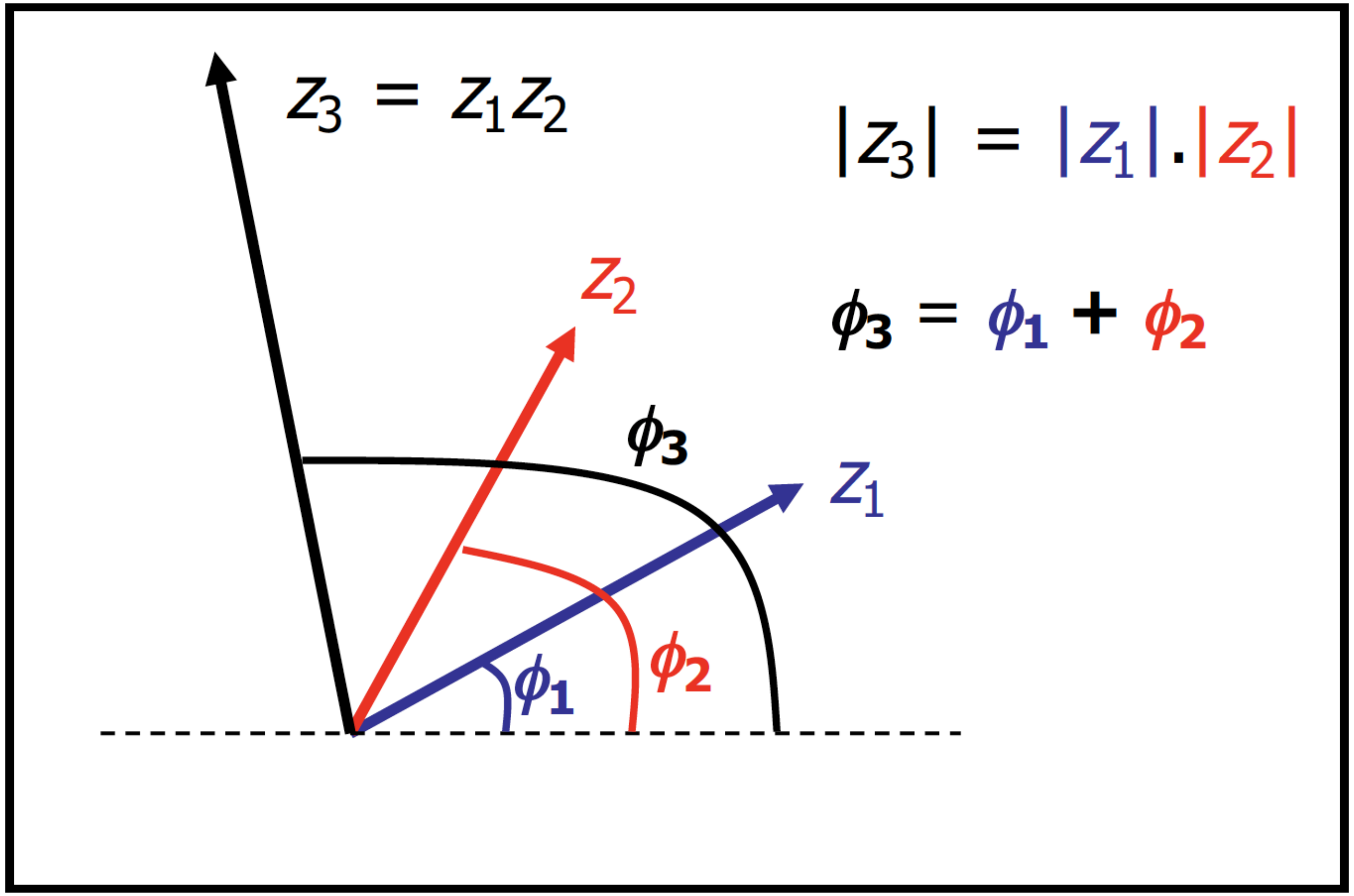


Using phasor diagram

- **Rule 1**
  - Each voltage and current is represented by a phasor drawn on the complex plane.
    - Length of phasor is proportional to the magnitude of the voltage or current
    - Argument of phasor represents the phasor angle
  - Resistor – current and voltage have same orientation
  - Inductor – current lags voltage by 90°
  - Capacitor – current leads voltage by 90°

- **Rule 2**
  - Summation, subtraction, multiplication and division are performed according to the rules of elementary complex number operation
    - Summation and subtraction – vectorial operation following the parallelogram law
    - Multiplication – product of the two magnitudes, sum of the two arguments (angles)
    - Division – quotient of the two magnitudes, difference pf the two arguments (angles)

Multiplication and division

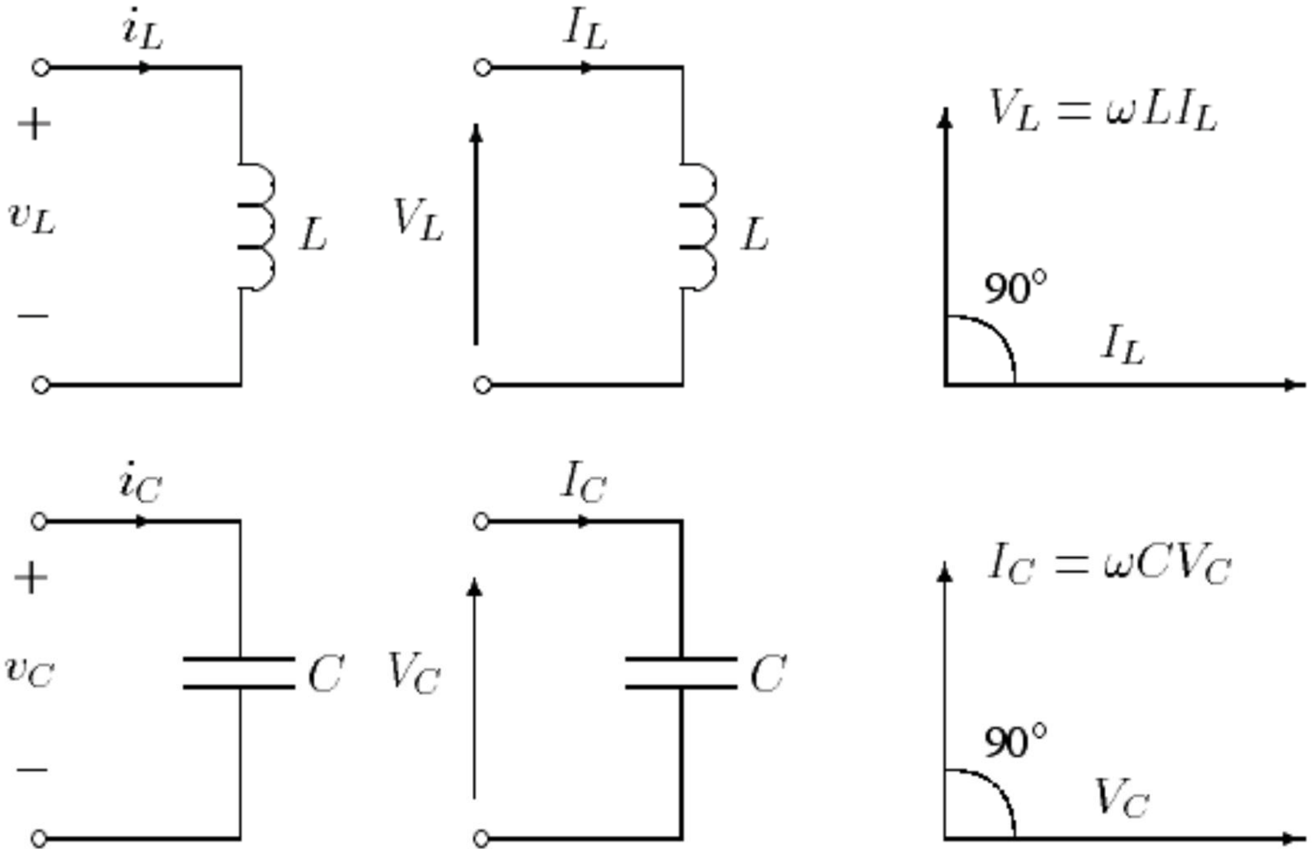


USEFUL TRICKS:

To rotate 90°
= multiply by *j*

To rotate –90°
= multiply by –*j*
= divide by *j*

Example:



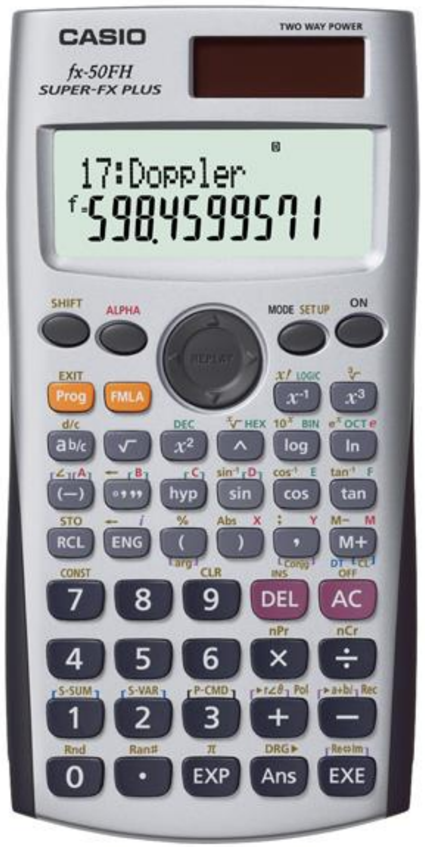
IMPEDANCE = V / I

$$j\omega L$$

$$\frac{1}{j\omega C} = -\frac{j}{\omega C}$$

Complex number calculations

Very easy to get results via Casio fx-50FH



■ Inputting Complex Numbers

Inputting Imaginary Numbers (i)

In the CMPLX Mode, the [ENG] key is used to input the imaginary number i. Use [ENG](i) when inputting complex numbers using rectangular coordinate format ($a+bi$).

Example: To input 2 + 3i

[2] [+] [3] [ENG](i) — CMPLX: 2+3 i

Inputting Complex Number Values Using Polar Coordinate Format

Complex numbers can also be input using polar coordinate format ($r \angle \theta$).

Example: To input 5 ∠ 30

[5] [SHIFT] [(–)](∠) [3] [0] — CMPLX: 5∠30

■ Complex Number Calculation Result Display

When a calculation produces a complex number result, R⇔I symbol turns on in the upper right corner of the display and the only the real part appears at first. To toggle the display between the real part and the imaginary part, press [SHIFT] [EXE] (Re⇔Im).

Example: To input 2 + 1i and display its calculation result

Before starting the calculation, you need to perform the following operation to change the complex number display setting to "$a+bi$" as shown below.

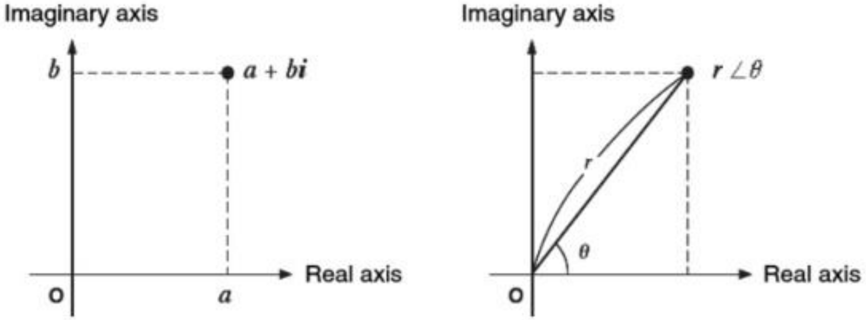
To select rectangular coordinate format: [SHIFT] [MODE] (SETUP) ▶ ▶ ▶ [1] ($a+bi$)

[2] [+] [ENG](i) [EXE] — CMPLX R⇔I: 2+i — 2.

Displays real part.

Default Complex Number Calculation Result Display Format

You can select either rectangular coordinate format or polar coordinate format for complex number calculation results.

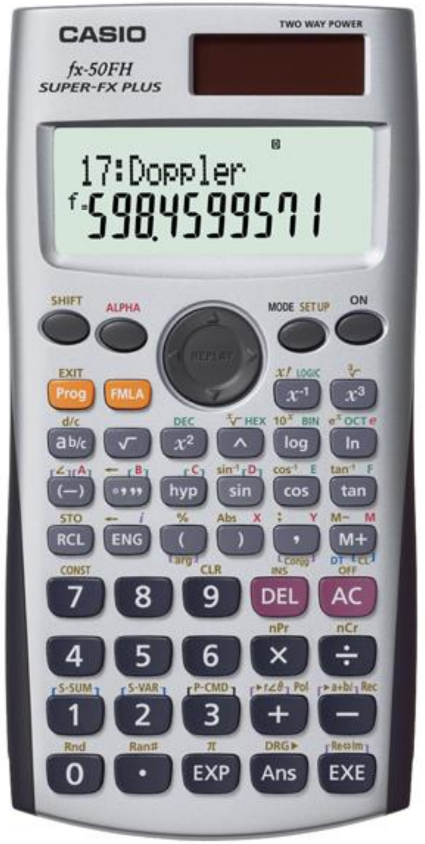


Rectangular Coordinates   Polar Coordinates

Use the setup screens to specify the default display format you want. For details, see "Specifying the Complex Number Display Format" (page 9).

Complex number calculations

Very easy to get results via Casio fx-50FH



■ Calculation Result Display Examples

Rectangular Coordinate Format ($a+bi$)

SHIFT MODE (SETUP) ▶ ▶ ▶ 1 ($a+bi$)

Example 1: $2 \times (\sqrt{3} + i) = 2\sqrt{3} + 2i = 3.464101615 + 2i$

2 × (√ 3) + ENG (i)) EXE → display: 2×(√(3)+i) 3.464101615

SHIFT EXE (Re⇔Im) → display: 2×(√(3)+i) 2. i

Example 2: $\sqrt{2} \angle 45 = 1 + 1i$ (Angle Unit: Deg)

√ 2) SHIFT (−) (∠) 4 5 EXE → display: √(2)∠45 1.

SHIFT EXE (Re⇔Im) → display: √(2)∠45 1. i

Polar Coordinate Format ($r\angle\theta$)

SHIFT MODE (SETUP) ▶ ▶ ▶ 2 ($r\angle\theta$)

Example 1: $2 \times (\sqrt{3} + i) = 2\sqrt{3} + 2i = 4 \angle 30$

2 × (√ 3) + ENG (i)) EXE → display: 2×(√(3)+i) 4.

SHIFT EXE (Re⇔Im) → display: 2×(√(3)+i) ∠ 30.

∠ symbol turns on during display of θ-value.

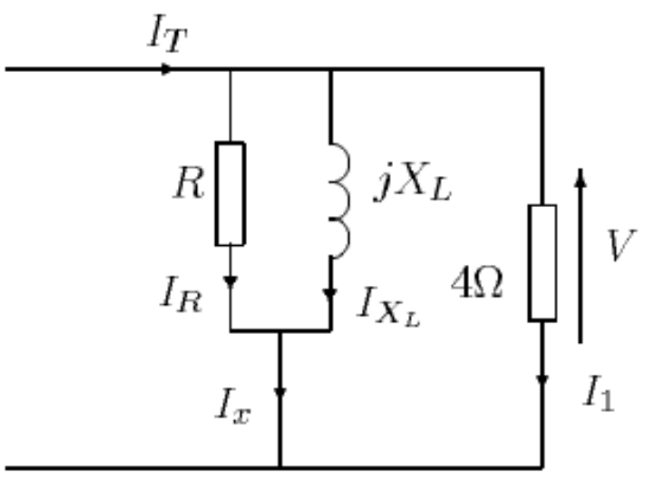
Example 2: $1 + 1i = 1.414213562 \angle 45$ (Angle Unit: Deg)

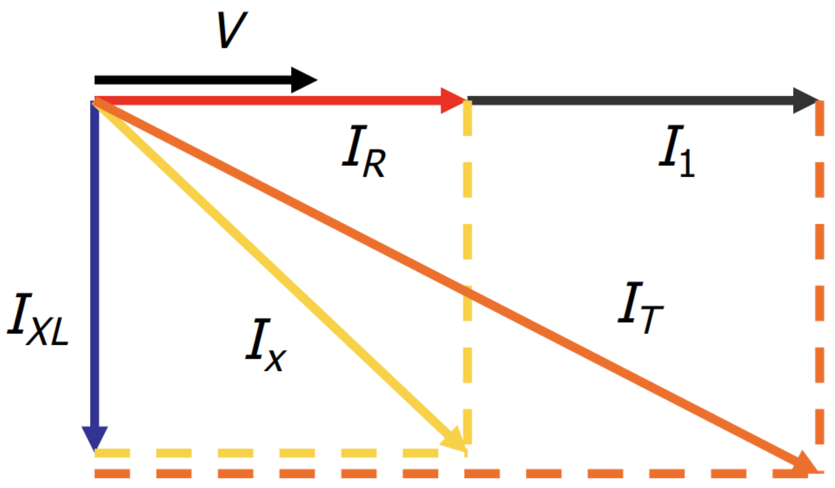
1 + 1 ENG (i) EXE → display: 1+1i 1.414213562

SHIFT EXE (Re⇔Im) → display: 1+1i ∠ 45.

Example:

Find $|R|$ and $|X_L|$ such that $|I_x|$= 18 A, $|I_1|$= 15 A and $|I_T|$= 30A





Since I_1 = 15 A, V = 60 V.

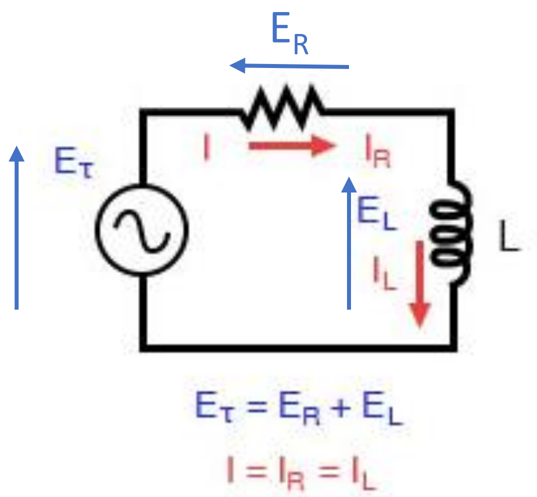
Hence,
$$|\text{Current in } R| = \frac{V}{R} = \frac{60}{R}$$
$$|\text{Current in } L| = \frac{V}{X_L} = \frac{60}{X_L}$$

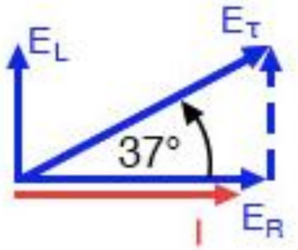
Applying Pythagoras theorem,
$$(|I_R| + |I_1|)^2 + |I_{X_L}|^2 = |I_T|^2$$
$$|I_{X_L}|^2 + |I_R|^2 = |I_x|^2$$

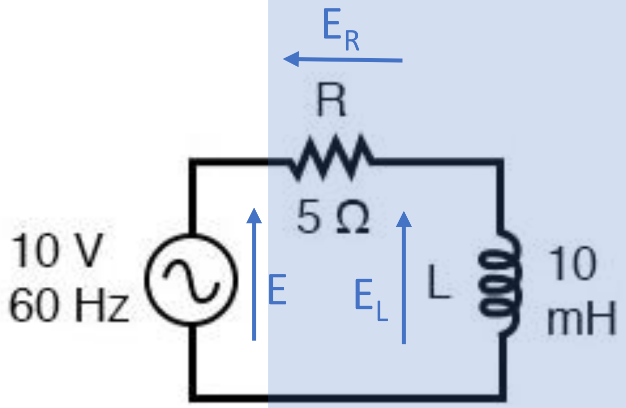
Hence,
$$\left(\frac{60}{R} + 15\right)^2 + \left(\frac{60}{X_L}\right)^2 = 30^2$$
$$\left(\frac{60}{X_L}\right)^2 + \left(\frac{60}{R}\right)^2 = 18^2$$

$|R|$ = 5.13 Ω $|X_L|$ = 4.36 Ω

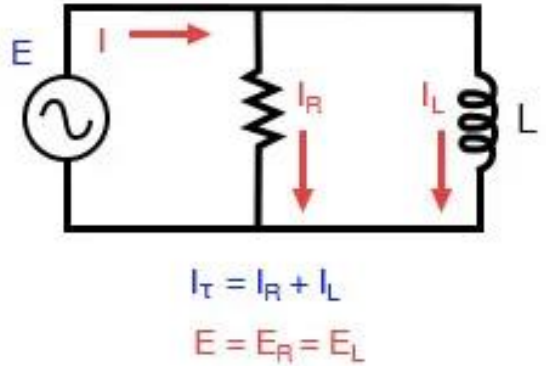
Impedance example

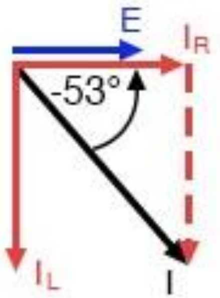


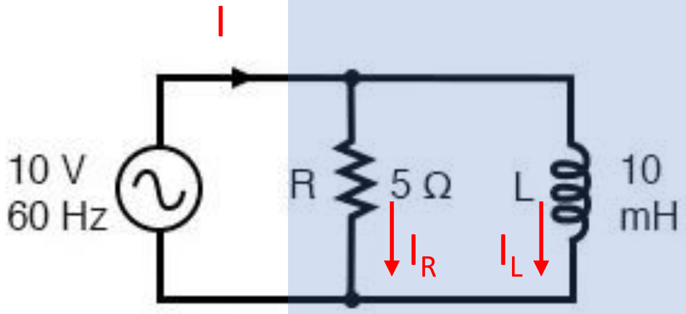




$$
\begin{aligned}
R &= 5\ \Omega \\
X_L &= (2\pi)(60)0.01\ \Omega \\
Z &= R + jX_L \\
&= 5 + j3.77\ \Omega \\
&= \sqrt{5^2 + 3.77^2}\angle 37^o\ \Omega \\
&= 6.26\angle 37^o\ \Omega
\end{aligned}
$$



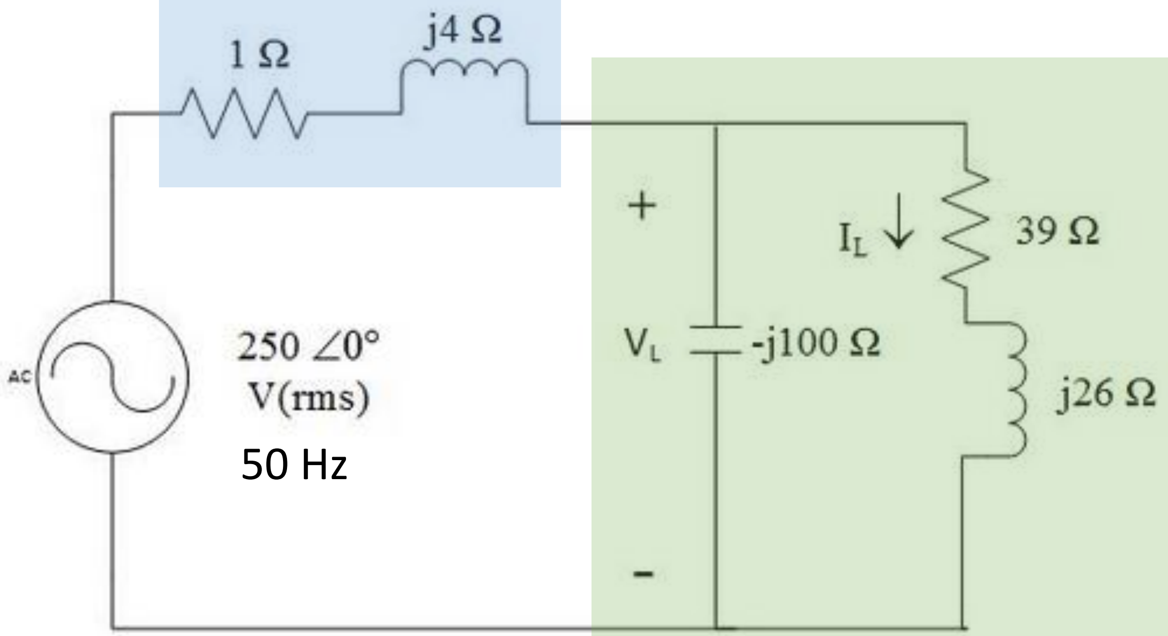




$$
\begin{aligned}
R &= 5\ \Omega \\
X_L &= (2\pi)(60)0.01\ \Omega \\
Z &= \frac{1}{\frac{1}{R} + \frac{1}{jX_L}} \\
&= \frac{j18.85}{(5 + j3.77)}\ \Omega \\
&= \frac{18.85\angle 90^o}{6.26\angle 37^o} = 3.01\angle 53^o\ \Omega
\end{aligned}
$$

Impedance example

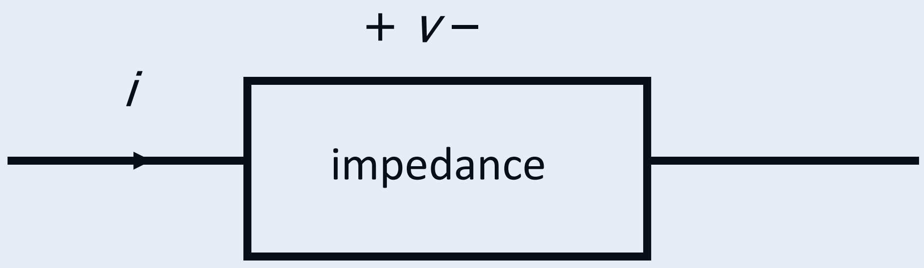
$$Z_{\text{LINE}} = R_l + jX_l\ \Omega = |Z_{\text{LINE}}|\angle\phi_{\text{LINE}}$$



$$Z_{\text{LOAD}} = R_L + jX_L\ \Omega$$
$$= |Z_{\text{LOAD}}|\angle\phi_{\text{LOAD}}\ ?\ ?$$

POWER

Impedance = Z = $X + jY$ = $|Z|\angle\phi = \dfrac{V}{I}$



Instantaneous power ***dissipation*** $p(t) = v(t)i(t)$

which is true for ALL kinds of elements.

NOTE: the sign convention! Positive current into the '+' terminal.

Average power

Normally when we refer to the power dissipation of an element, it is always the ***average*** power. (Active power or real power)

$$P = P_{\mathrm{av}} = \frac{1}{T}\int_0^T v(t)i(t)dt$$

where *T* is the period of repetition of the power waveform. For sinusoidal voltage and current, this is

$$P = \frac{1}{T}\int_0^T \hat{v}\hat{i}\sin^2 \omega t dt = \frac{1}{2\pi}\int_0^{2\pi} \hat{v}\hat{i}\sin^2 \theta d\theta = \frac{1}{2}\hat{v}\hat{i}$$

Power in resistor

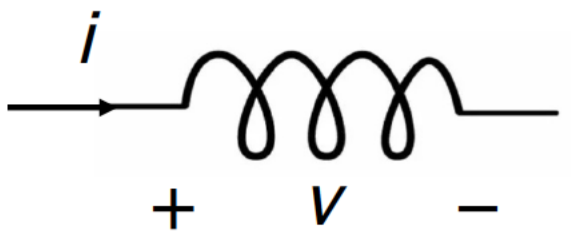
For resistor R, $v(t) = i(t)R$

Power dissipation is $P = \frac{\hat{v}^2}{2R} \text{ OR } \frac{\hat{i}^2 R}{2}$

If V and I are the ***root-mean-square values*** of voltage and current, then

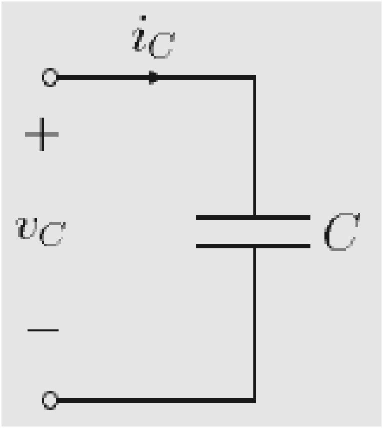
$$P = \frac{V^2}{R} \text{ OR } I^2 R$$

Power in inductor and capacitor



$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$i_C = C\frac{dv_C}{dt}$$



For L or C, the voltage and current are always 90º out of phase. Thus,

$$P = \frac{1}{2\pi}\int_0^{2\pi} \hat{v}\hat{i}\sin\theta\cos\theta d\theta = \frac{1}{2\pi}\int_0^{2\pi} \hat{v}\hat{i}\frac{\sin 2\theta}{2}d\theta = 0$$

ZERO POWER DISSIPATION

Reactive power

Although L and C do not dissipate power, they do have instantaneous power, i.e., "give" and "take" power continuously. The magnitude of this instantaneous power is not zero!

For L and C, we define Reactive Power Q (unit: var) as

$$Q = V_{\mathrm{rms}} I_{\mathrm{rms}} = \begin{cases} \omega L I_{\mathrm{rms}}^2 & \text{for inductor} \\ \omega C V_{\mathrm{rms}}^2 & \text{for capacitor} \end{cases}$$

$$Q = I_{\mathrm{rms}}^2 X = \frac{V_{\mathrm{rms}}^2}{X}$$

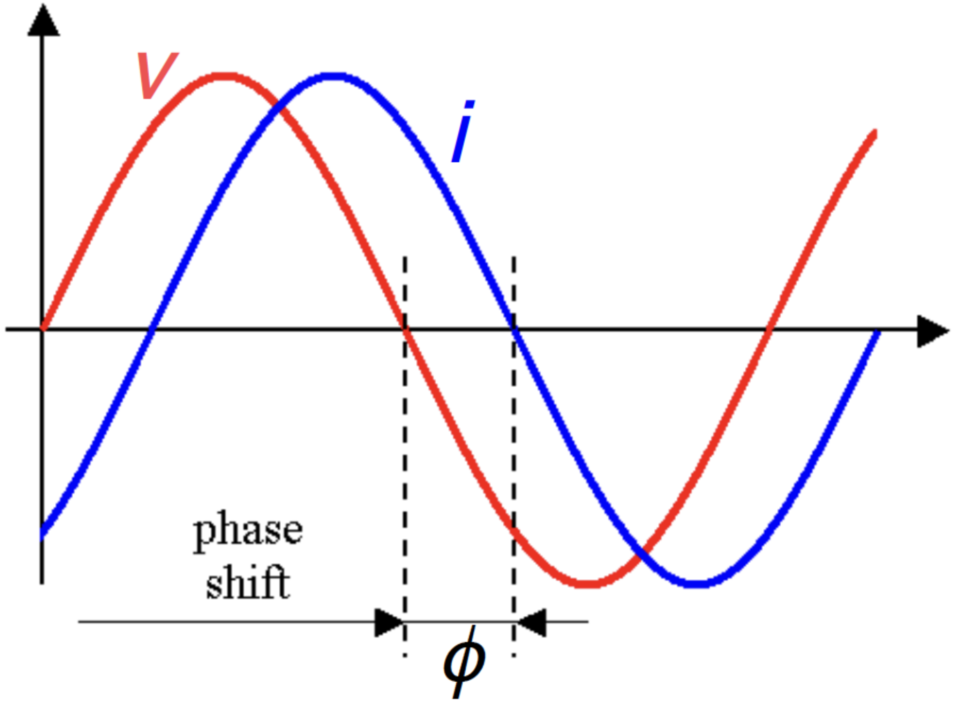
X is called REACTANCE (same unit as resistance)

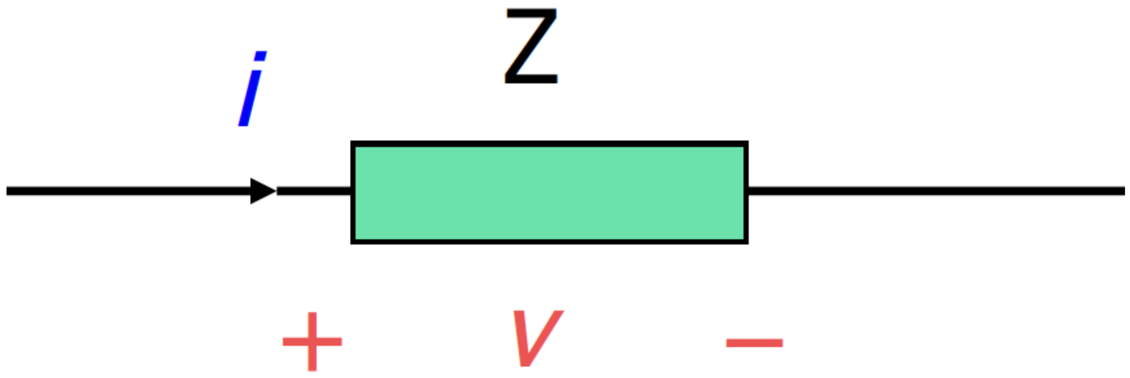
Active and reactive power

In general, suppose *v* leads *i* by an angle ϕ.

$$v(t) = \hat{v} \sin \omega t$$

$$i(t) = \hat{i} \sin(\omega t - \phi)$$

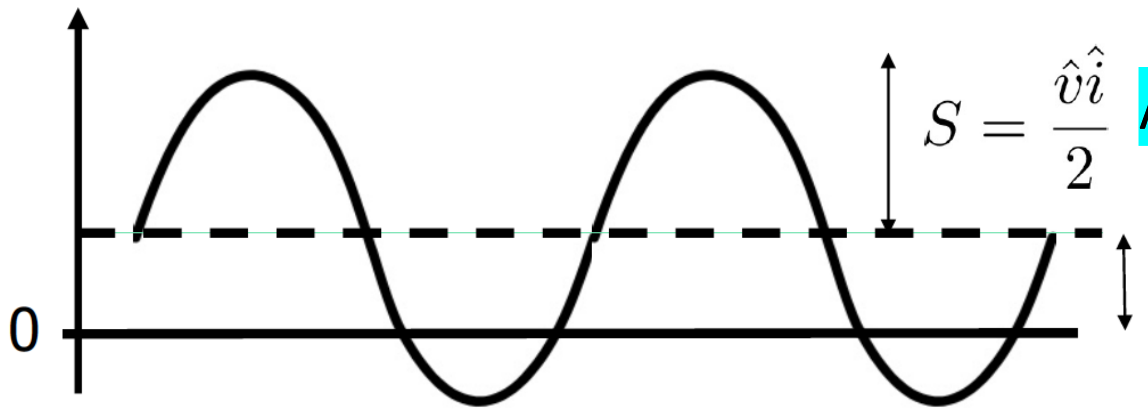




Instantaneous power

Instantaneous power dissipated in *Z* is

$$p(t) = v(t)i(t) = \hat{v}\hat{i}\sin\omega t \sin(\omega t - \phi) = \frac{\hat{v}\hat{i}}{2}[\cos\phi - \cos(2\omega t - \phi)]$$



$$S = \frac{\hat{v}\hat{i}}{2}$$

APPARENT POWER

$$P = \frac{\hat{v}\hat{i}}{2}\cos\phi$$

REAL POWER or ACTIVE POWER

The average power P is also called real power or active power (unit: W).

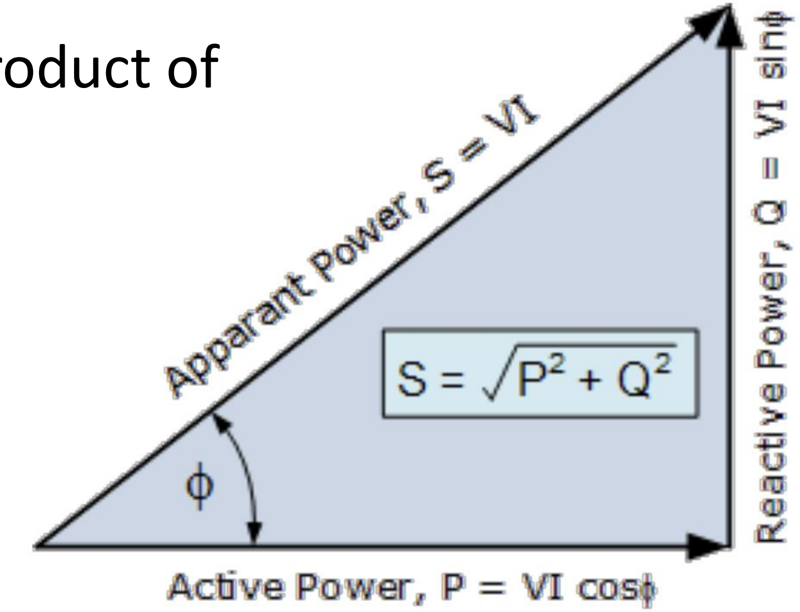
Power triangle

In complex number representation, *S* is the dot product of *V* and the *conjugate* of *I.*

$$S = V.I^* = |V|.|I|\angle\phi$$

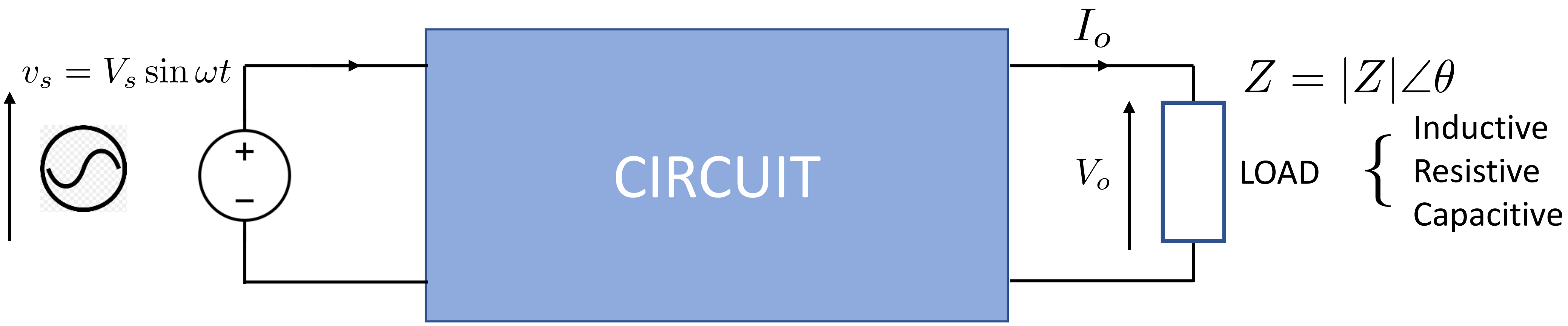
$$P = \Re\{S\} = |V|.|I|\cos\phi$$

$$Q = \Im\{S\} = |V|.|I|\sin\phi$$



Single phase power circuits

One main sinusoidal source



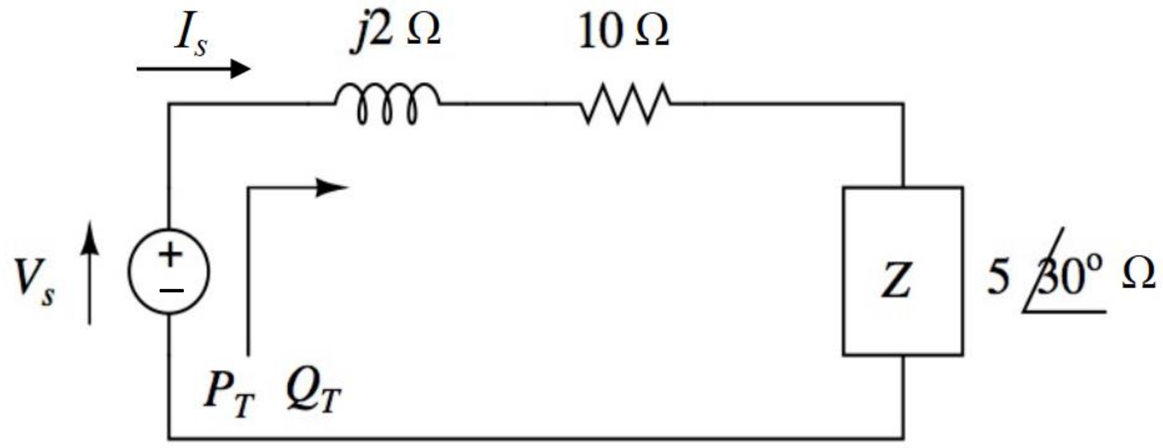
$$Z = |Z|\angle\theta \qquad Z = \frac{V_o}{I_o}$$

The angle θ is exactly the phase difference between the output voltage and current.

Power concept

Suppose V_s is 10 V rms.
Find the current I_s.

- Very tedious for circuit analysis.
- But much easier by power concept.



TOTAL power

$$P_T = |I_s|^2(10 + 5\cos 30^o) = 14.33|I_s|^2 \text{ W}$$
$$Q_T = |I_s|^2(2 + 5\sin 30^o) = 4.5|I_s|^2 \text{ var}$$

Therefore, $|S_T| = \sqrt{|P_T|^2 + |Q_T|^2|} = 15.02|I_s|^2 \text{ VA}$

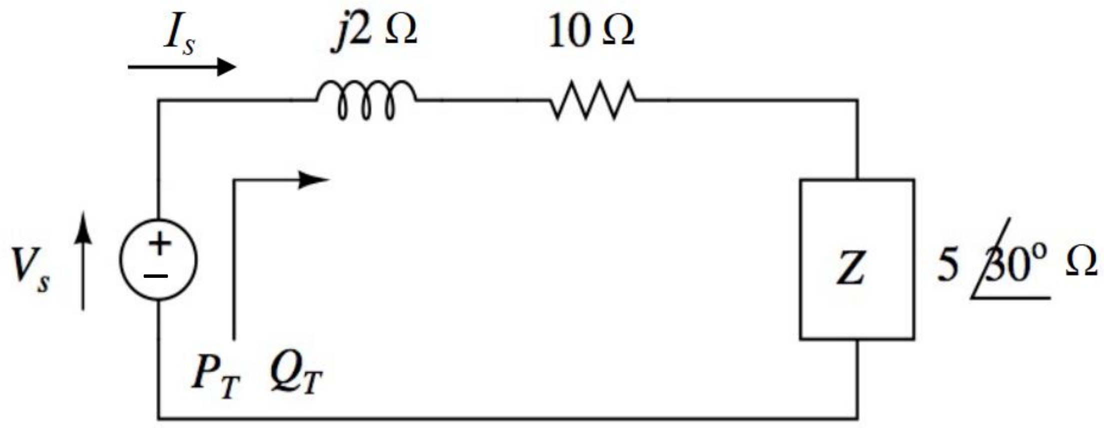
Power concept

The magnitude of S_T is $|S_T| = |V_s|.|I_s|$
So,

$$|I_s| = \frac{10}{15.02} = 0.665 \text{ A}$$

Phase angle between V_s and I_s is ϕ.

$$\phi = \arctan\left(\frac{Q_T}{P_T}\right) = 17.43^o$$



Therefore, $I_s = 0.665\angle -17.43^o \text{ A}$

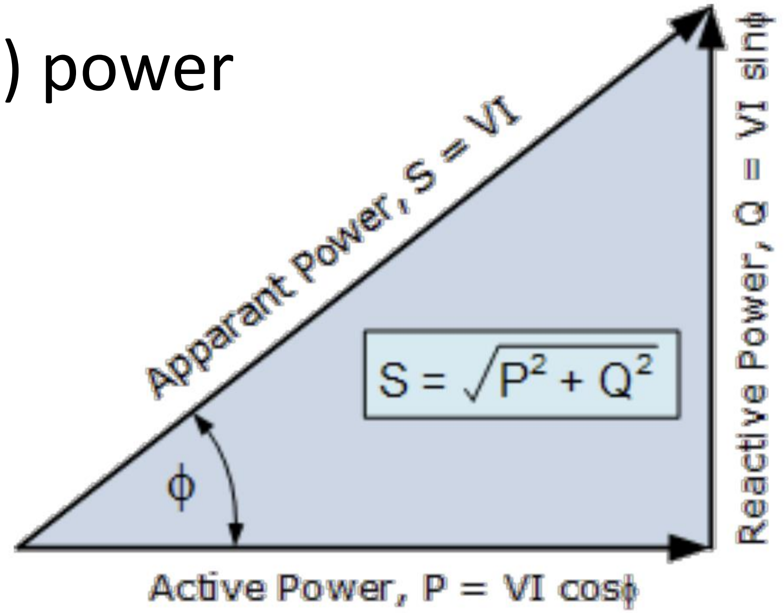
Power factor

- Measure of the relative content of real (workable) power

$$\text{p.f.} = \frac{P}{S}$$

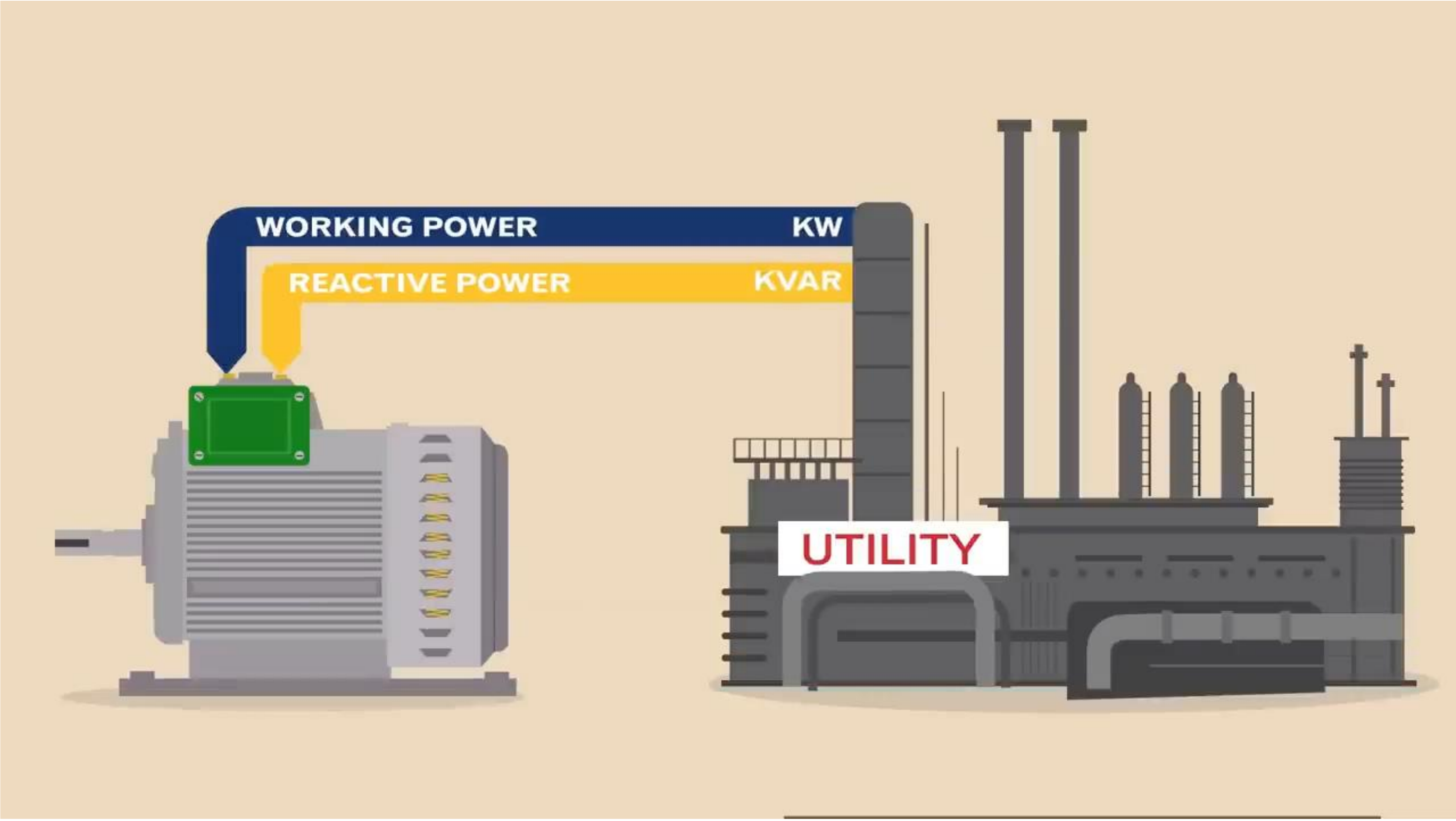
- For linear circuits:

$$\text{p.f.} = \cos\phi$$



- When p.f. = 1, there is zero reactive power input.
- **WHY IS IT IMPORTANT TO KEEP p.f. HIGH (CLOSE TO 1)?**
  - **To reduce reactive power which does not do any real work but increases the current magnitude, hence wasting resources, e.g., thicker cables.**

What is Power Factor?



https://www.youtube.com/watch?v=NF4VRKa7LSM&ab_channel=RapidPowerManagement

Disadvantages of low power factor:

A power factor less than unity results in the following disadvantages:

- Large KVA rating of equipment.

  $kVA = kW/(\cos\phi)$

KVA rating of the equipment is inversely proportional to power factor.

- Greater conductor size.

To transmit or distribute a fixed amount of power at constant voltage, the conductor will have to carry more current at low power factor.

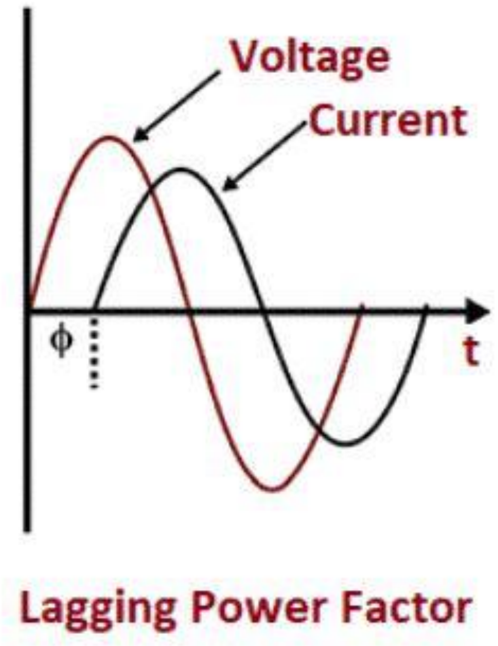
- Large copper losses.
- Poor voltage regulation.

The large current at low lagging power factor causes greater voltage drops in alternators, transformers, transmission lines and distributors. This results in the decreased voltage available at the supply end.

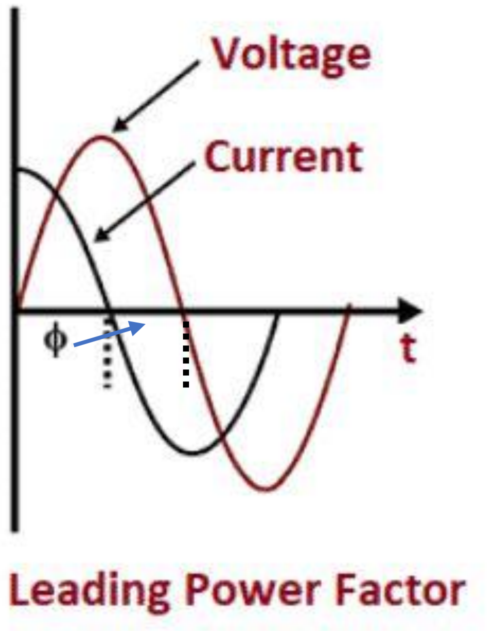
Power factor from waveforms

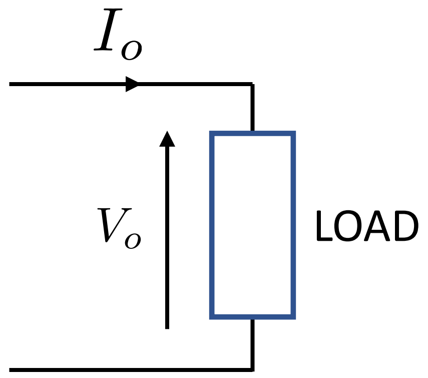
$$\text{p.f.} = \cos\phi$$

inductive



capacitive

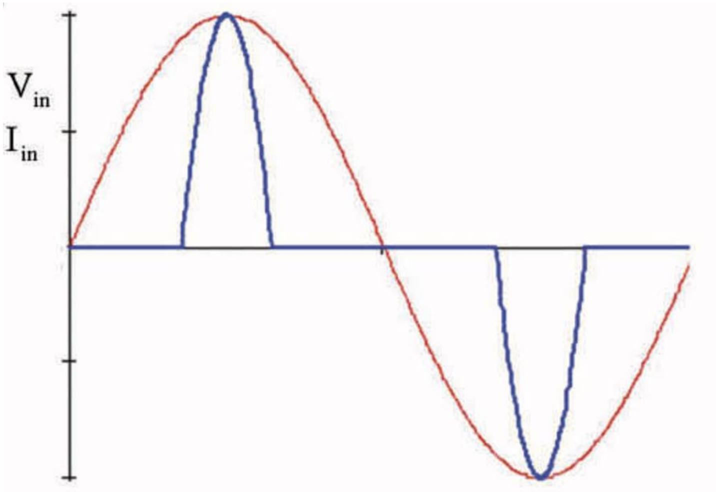




More advanced definition of p.f.

Current is not a sinusoid but has harmonics!

Suppose $I_{1,\text{rms}}$ is the fundamental magnitude.



$$\text{distortion factor} = \frac{I_{1,\text{rms}}}{I_{\text{T,rms}}} \longleftarrow \text{p.f.}$$

$$= \frac{I_{1,\text{rms}}}{\sqrt{I_{1,\text{rms}}^2 + I_{2,\text{rms}}^2 + \cdots + I_{\text{N,rms}}^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{I_{2,\text{rms}}^2 + \cdots + I_{\text{N,rms}}^2}{I_{1,\text{rms}}^2}}} = \frac{1}{\sqrt{1 + \text{THD}^2}}$$

[Total harmonic distortion (THD)]

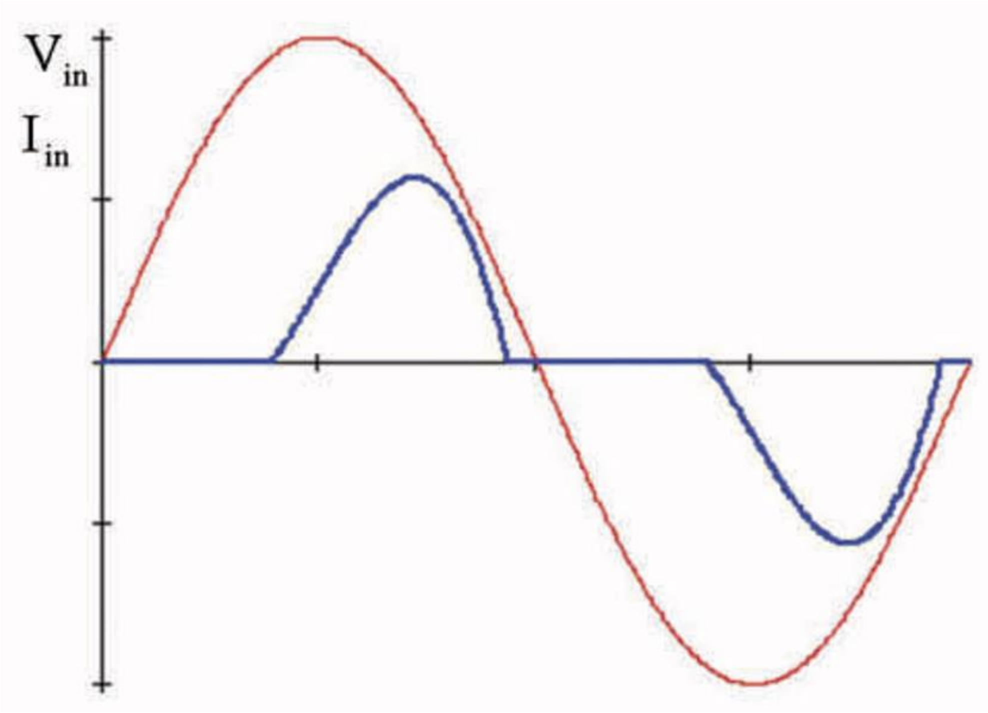
Even though the phase angle difference is ZERO, the p.f. is less than 1 and is equal to the distortion factor.

More advanced definition of p.f.

$$\text{p.f.} = \frac{\text{Active Power}}{V_{\text{rms}} I_{\text{rms}}}$$

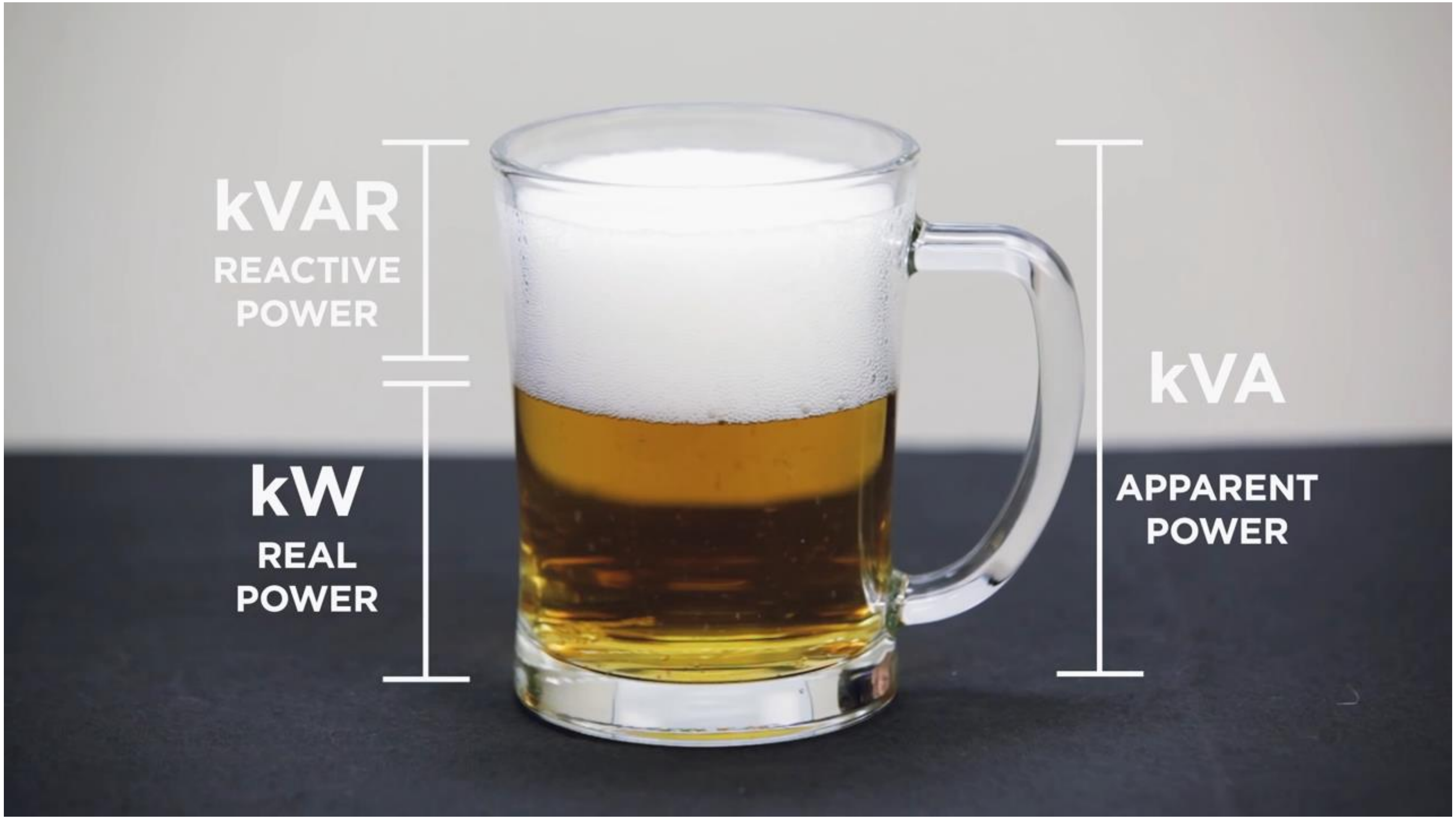
$$= \cos\phi \times \text{distortion factor}$$

$$= \cos\phi \times \frac{1}{\sqrt{1 + \text{THD}^2}}$$



Both the angle displacement and distortion will affect the power factor.

Difference between real, reactive and apparent power



https://www.youtube.com/watch?v=8zRKzRRDkm8&ab_channel=TitanPowerInc

Example: real and reactive power

A single-phase voltage source with $V = 100\angle 130°$ volts delivers a current $I = 10\angle 10°$ A, which leaves the positive terminal of the source. Calculate the source real and reactive power, and state whether the source delivers or absorbs each of these.

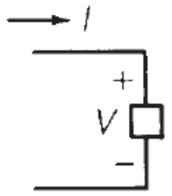
SOLUTION

Since ***I*** leaves the positive terminal of the source, the generator convention is assumed, and the complex power delivered is

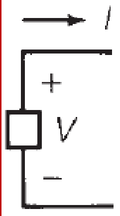
$$S = VI^* = [100\angle 130°][10\angle 10°]^*$$
$$S = 1000\angle 120° = -500 + j866$$
$$\mathrm{P} = \mathrm{Re}[S] = -500 \quad \mathrm{W}$$
$$\mathrm{Q} = \mathrm{Im}[S] = +866 \quad \mathrm{var}$$

$$\boxed{\begin{aligned} S = VI^* &= [\mathrm{V}\angle \delta][\mathrm{I}\angle \beta]^* = \mathrm{VI}\angle \delta - \beta \\ &= \mathrm{VI}\cos(\delta - \beta) + j\mathrm{VI}\sin(\delta - \beta) \end{aligned}}$$

where *Im* denotes ‘‘imaginary part of.’’ The source absorbs 500 W and delivers 866 var.



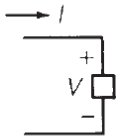
(a) *Load convention.* Current *enters* positive terminal of circuit element. If P is positive, then positive real power is *absorbed.* If Q is positive, then positive reactive power is *absorbed.* If P (Q) is negative, then positive real (reactive) power is *delivered.*



(b) *Generator convention.* Current *leaves* positive terminal of the circuit element. If P is positive, then positive real power is *delivered.* If Q is positive, then positive reactive power is *delivered.* If P (Q) is negative, then positive real (reactive) power is *absorbed.*

Example: power factor correction

A single-phase source delivers 100 kW to a load operating at a power factor of 0.8 lagging. Calculate the reactive power to be delivered by a capacitor connected in parallel with the load in order to raise the source power factor to 0.95 lagging. Also draw the power triangle for the source and load. Assume that the source voltage is constant, and neglect the line impedance between the source and load.

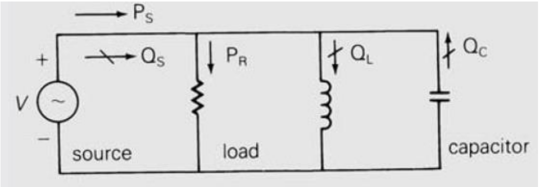


(a) *Load convention.* Current *enters* positive terminal of circuit element. If P is positive, then positive real power is *absorbed.* If Q is positive, then positive reactive power is *absorbed.* If P (Q) is negative, then positive real (reactive) power is *delivered.*

Load convention is the most commonly used.

SOLUTION

The real power P = P_S = P_R delivered by the source and absorbed by the load is not changed when the capacitor is connected in parallel with the load, since the capacitor delivers only reactive power QC. For the load, the power factor angle, reactive power absorbed, and apparent power are



$$\theta_L = \cos^{-1}(0.8) = 36.87^\circ$$

$$Q_L = P\tan\theta_L = 100\tan(36.87^\circ) = 75 \quad \text{kvar}$$

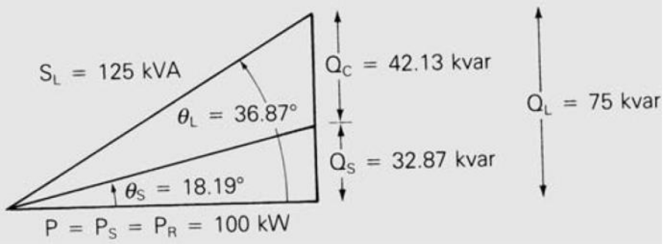
$$S_L = \frac{P}{\cos\theta_L} = 125 \quad \text{kVA}$$

After the capacitor is connected, the power factor angle, reactive power delivered, and apparent power of the source are

$$\theta_S = \cos^{-1}(0.95) = 18.19^\circ$$

$$Q_S = P\tan\theta_S = 100\tan(18.19^\circ) = 32.87 \quad \text{kvar}$$

$$S_S = \frac{P}{\cos\theta_S} = \frac{100}{0.95} = 105.3 \quad \text{kVA}$$



The capacitor delivers

$$Q_C = Q_L - Q_S = 75 - 32.87 = 42.13 \quad \text{kvar}$$

Power factor correction

Normally, the power factor of the whole load on a large generating station is in the region of 0.8 to 0.9. However, sometimes it is lower and in such case, it is generally desirable to take special steps to improve the power factor. This can be achieved by the following equipment:

- Static capacitors
- Synchronous condenser
- Phase advancers

Power factor correction: Static capacitor

- A capacitor is a device that stores electric charge Q and energy U *(W).*

- Can be found in many systems:
  - Key components for electric circuits/systems: L, C, R; Battery, Generator, power line, etc..
  - Energy-storing devices
  - To eliminate sparking in automobile ignition systems
  - Cell phones, computers, iPads, TVs, air conditioners, etc.
  - Radio receivers
  - Filters in power supplies
  - Resonant LC circuit

Power factor correction: Static capacitor

The power factor can be improved by connecting capacitors in parallel with the equipment operating at lagging power factor. The capacitor draws a leading current and partly or completely neutralizes the lagging reactive component of load current.

Advantages:

- Low losses.
- Little maintenance
- Easy installation
- Working under ordinary atmospheric conditions.

Disadvantages:

- Short service life. (8-10 years)
- Easily damaged if voltage exceeds the rated value.
- Uneconomical reparation.

Power factor correction: Synchronous condenser

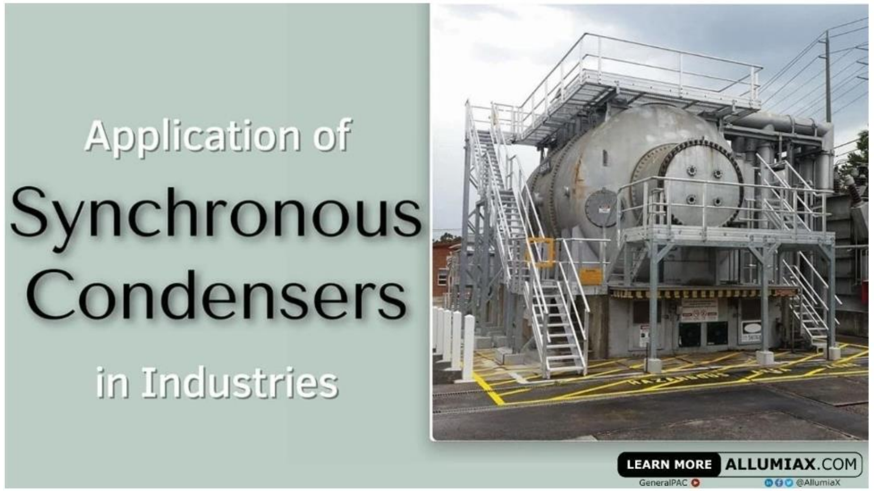
A synchronous motor takes a leading current when over-excited and, therefore, behaves as a capacitor. An over-excited synchronous motor running on no load is known as a synchronous condenser. When such a machine is connected in parallel with the supply, it takes a leading current which partly neutralizes the lagging reactive component of the load. Thus, the power factor is improved.

Advantages:

- The magnitude of current can be changed by any amount.
- High thermal stability to short circuit currents.
- The faults can be removed easily.

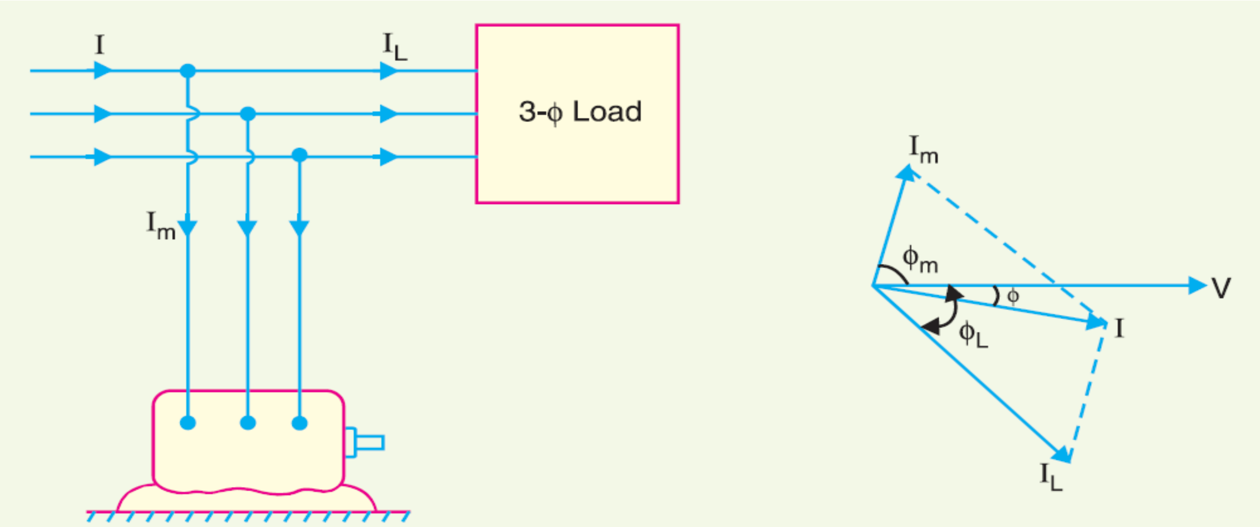
Disadvantages:

- Considerable losses
- Maintenance cost is high.
- Auxiliary equipment is required for self start.



Power factor correction: Synchronous condenser

Figure below shows the power factor improvement by synchronous condenser method. The 3ϕ load takes current I_L at low lagging power factor cos ϕ_L. The synchronous condenser takes a current I_m which leads the voltage by an angle ϕ_m*. The resultant current I is the phasor sum of I_m and I_L and lags behind the voltage by an angle ϕ. It is clear that ϕ is less than ϕ_L so that cos ϕ is greater than cos ϕ_L. Thus the power factor is increased from cos ϕ_L to cos ϕ. Synchronous condensers are generally used at major bulk supply substations for power factor improvement.



Power factor correction: Phase advancer

Phase advancers are used to improve the power factor of induction motors. The low power factor of an induction motor is because that its stator winding draws exciting current which lags be-hind the supply voltage by 90^{o}. If the exciting ampere turns can be provided from some other AC source, then the stator winding will be relieved of exciting current and the power factor of the motor can be improved. This job is accomplished by the phase advancer which is simply an AC exciter. The phase advancer is mounted on the same shaft as the main motor and is connected in the rotor circuit of the motor. It provides exciting ampere turns to the rotor circuit at slip frequency. By providing more ampere turns than required, the induction motor can be made to operate on leading power factor like an over-excited synchronous motor.

Advantages:

- As the exciting ampere turns are sup-plied at slip frequency, therefore, lagging kVAR drawn by the motor are considerably reduced.
- phase advancer can be conveniently used where the use of synchronous motors is inadmissible.

Disadvantages:

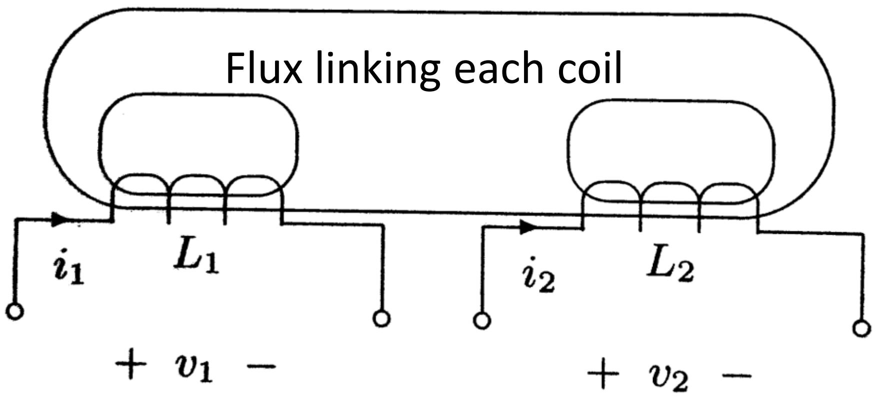
- Not economical for motors below 200 H.P.

Electrical transformer

permits changing AC voltage level

Transformer concept – coupled inductors

Inductance is the tendency of an electrical conductor to oppose a change in the electric current flowing through it.



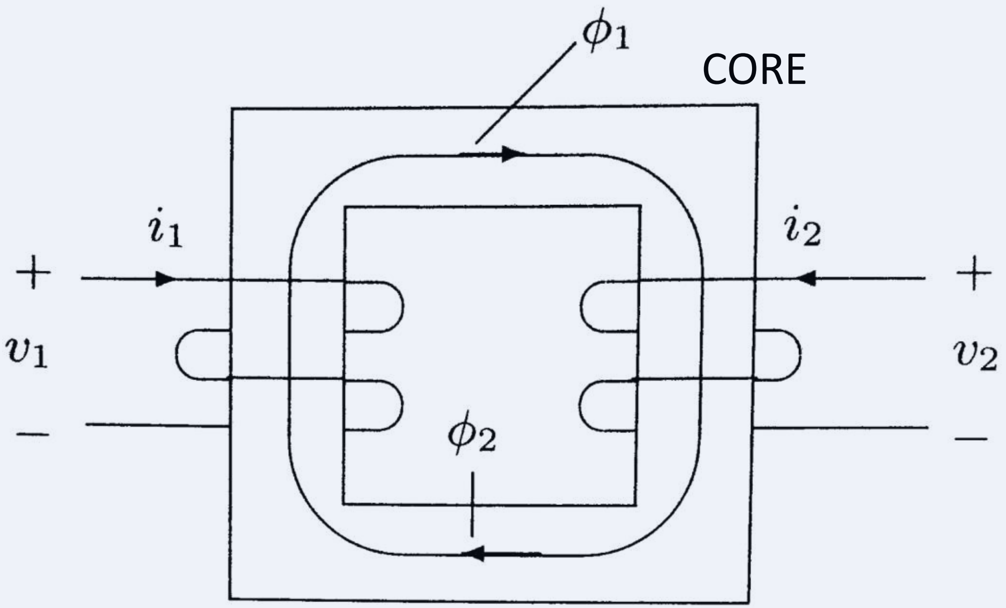
$$v_1 = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

$$v_2 = L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

L_{11} and L_{22} : self inductance

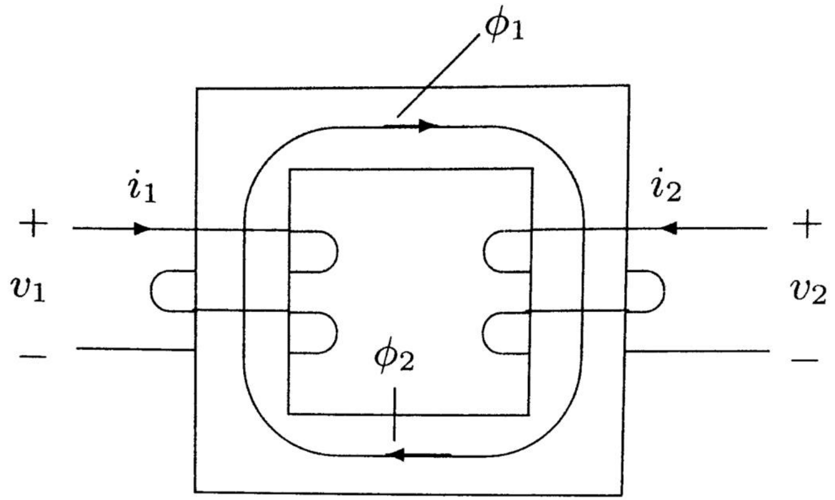
$L_{12} = L_{21} = M$: mutual inductance

Physical construction with a magnetic core

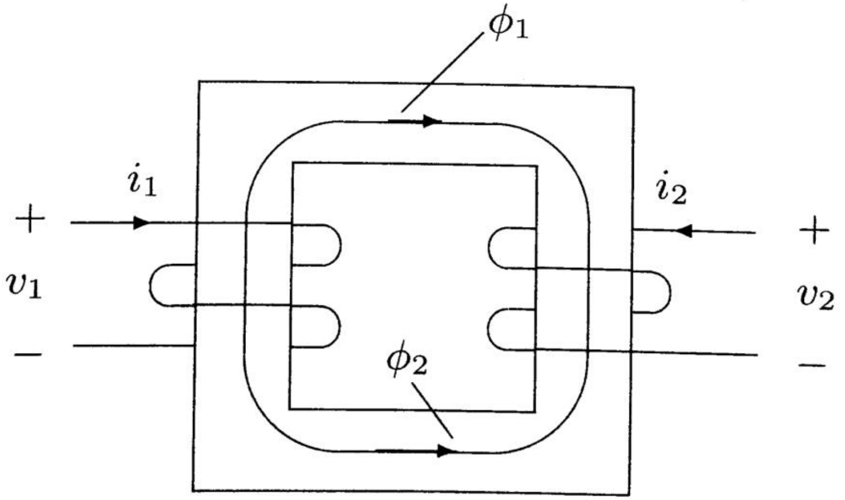


Fluxes are produced in the same "sense" by the two coils

Transformer polarity



Fluxes in same direction



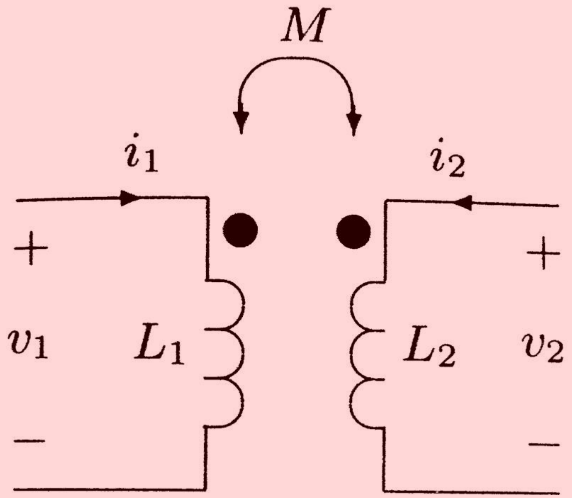
Fluxes in opposite directions

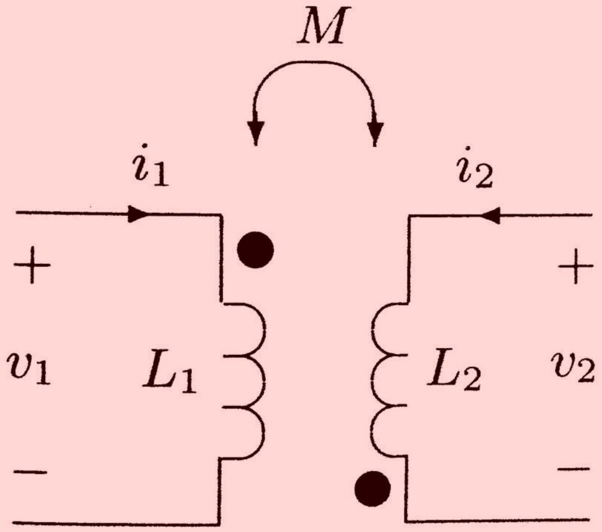
Write L_1 and L_2 for simplicity

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$
$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$
$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

DOT CONVENTION





Properties

It can be shown that $L_1 L_2 > M^2$

We define coupling coefficient *k*

$$k = \sqrt{\frac{M^2}{L_1 L_2}}$$

The value of *k* reflects how tight the two coils are coupled. $k < 1$.

Open-circuit secondary $i_2 = 0$

$$v_1 = L_1 \frac{di_1}{dt} + \cancel{M \frac{di_2}{dt}}$$

$$v_2 = M \frac{di_1}{dt} + \cancel{L_2 \frac{di_2}{dt}}$$

$$\frac{v_2}{v_1} = \frac{M}{L_1} = n$$

Short-circuit secondary $v_2 = 0$

$$0 = M i_1 + L_2 i_2$$

$$\frac{i_2}{i_1} = -\frac{M}{L_2} = -\frac{1}{n}$$

Transformer: Basic Calculation

Ideal case: $M^2 = L_1L_2$ or $k = 1$

Perfect coupling

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = \frac{M}{L_1}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right)$$

$$\Rightarrow v_2 = \frac{M}{L_1}v_1$$

The two coils have same flux linkage per turn under perfect coupling.

$$v_1 = N_1\frac{d}{dt}\Phi$$

$$v_2 = N_2\frac{d}{dt}\Phi$$

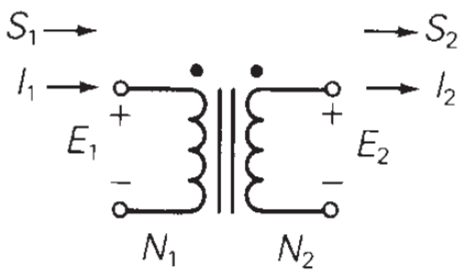
$$\Rightarrow v_2 = \frac{N_2}{N1}v_1$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = \frac{M}{L_1} = \frac{L_2}{M}$$

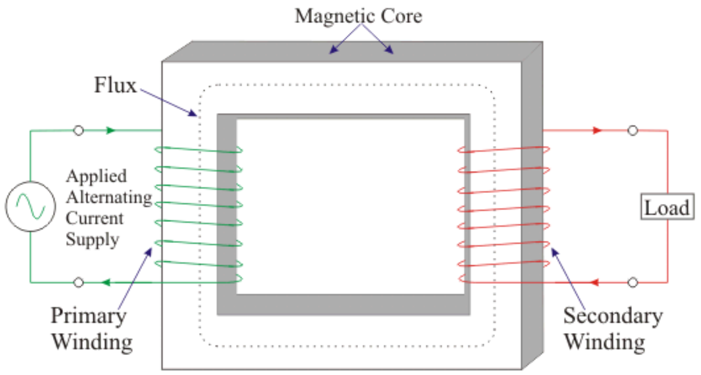
Under perfect coupling

Transformer: summary

single-phase two
winding-transformer



$$a_t = \frac{N_1}{N_2}$$



Single Phase Transformer

The turns ratio a_t is defined as follows:

$$a_t = \frac{N_1}{N_2} \qquad \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$E_1 = \left(\frac{N_1}{N_2}\right) E_2 = a_t E_2$$

$$I_1 = \left(\frac{N_2}{N_1}\right) I_2 = \frac{I_2}{a_t}$$

$$S_1 = E_1 I_1^* = (a_t E_2)\left(\frac{I_2}{a_t}\right)^* = E_2 I_2^* = S_2$$

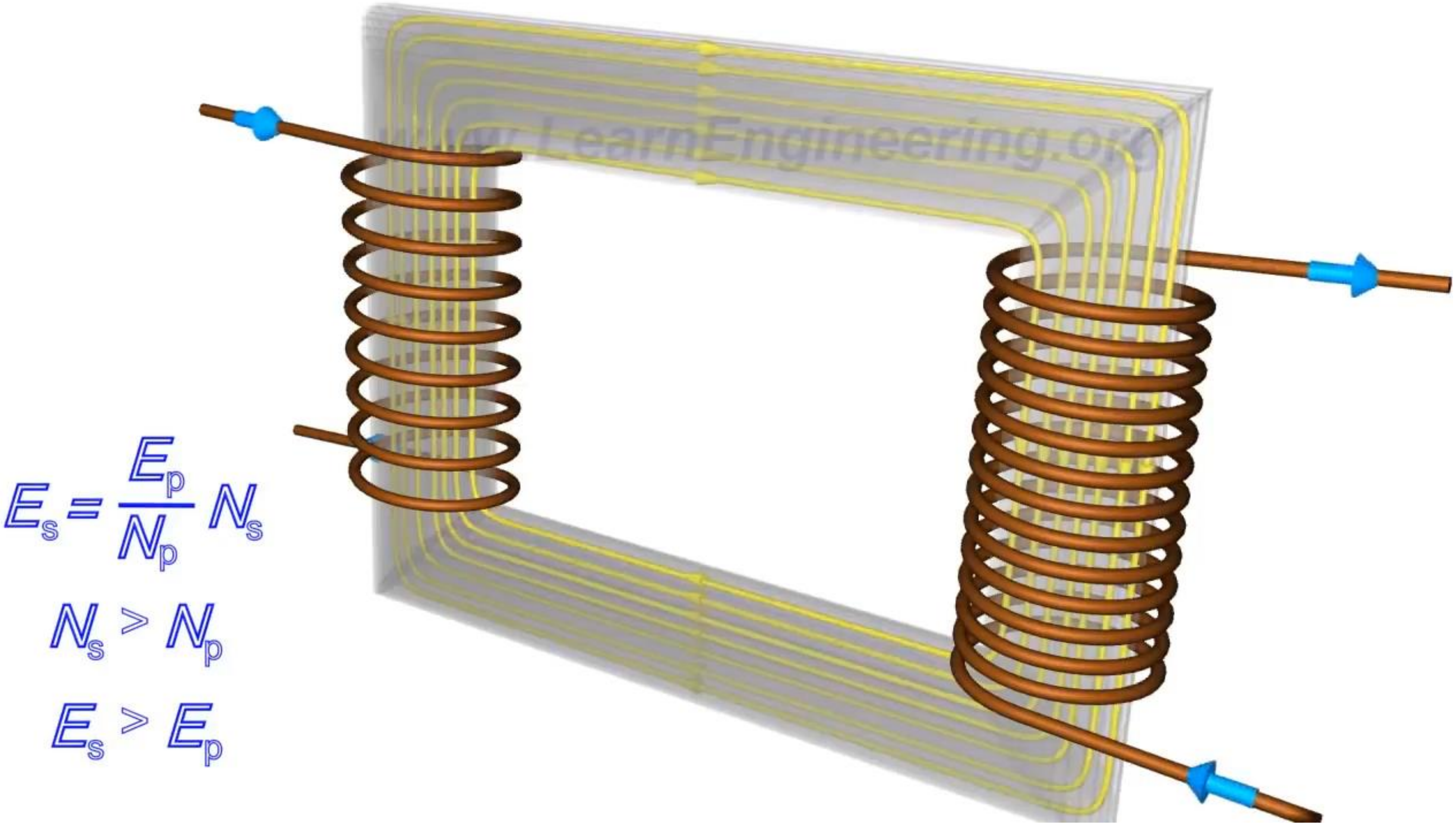
If an impedance Z_2 is connected across winding 2

$$Z_2 = \frac{E_2}{I_2}$$

This impedance, when measured from winding 1, is

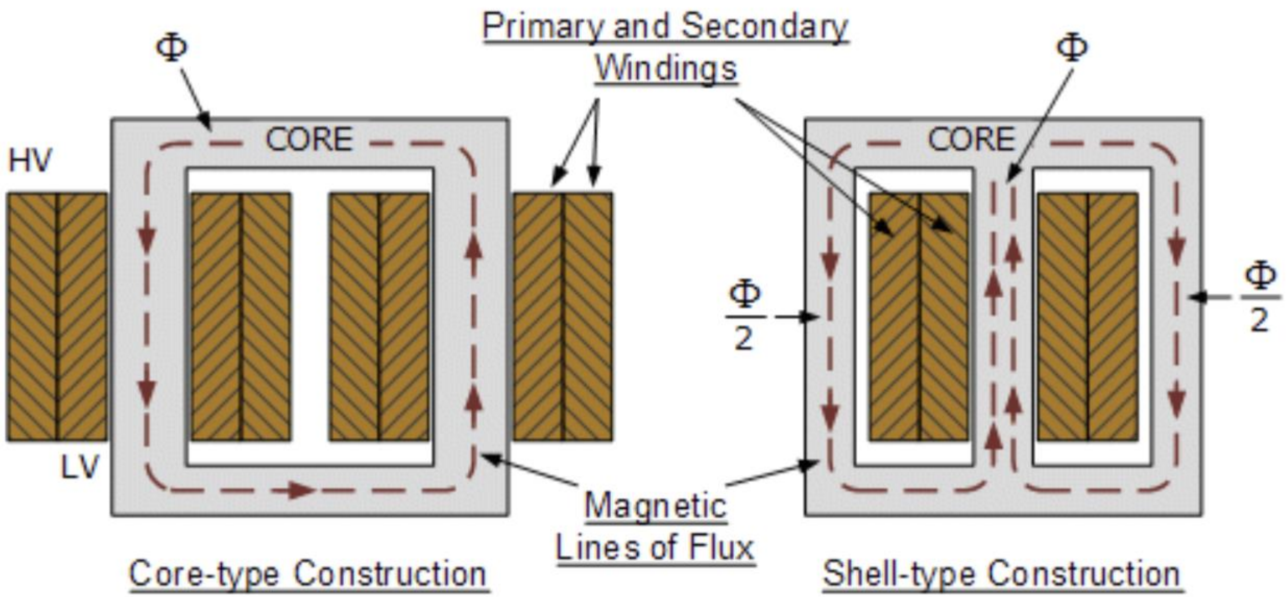
$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{I_2 / a_t} = a_t^2 Z_2 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

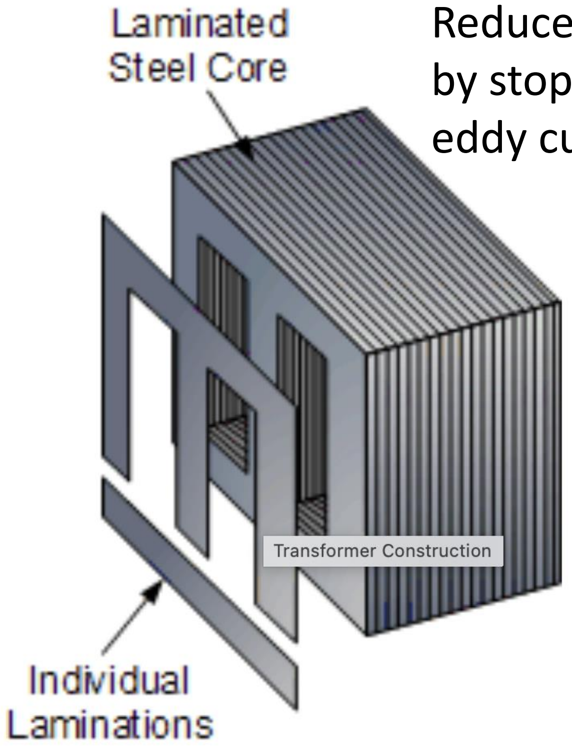
How does a transformer work



https://www.youtube.com/watch?v=vh_aCAHThTQ&ab_channel=Lesics

Construction





Reduce losses by stopping eddy currents

Winding schemes, core materials, core lamination would affect efficiency of the transformer.

Core loss (in magnetic core)

Copper loss (in metal wires)

https://www.electronics-tutorials.ws/transformer/transformer-construction.html

Core materials

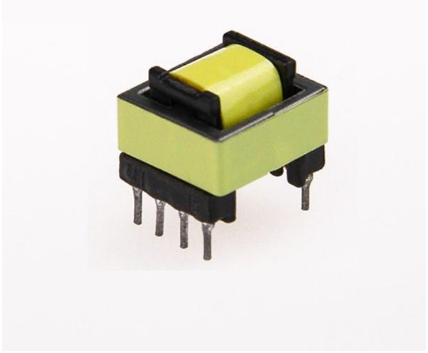
- Solid iron
  - Low frequency, high power, highly permeable, lossy due to eddy currents
- Amorphous steel
  - Medium frequency, reduced core losses due to layers of metallic tapes
- Ferrites
  - High frequency, less lossy, hysteresis/saturation issues

Other materials: Silicon steel, carbinyl iron, etc. have different properties for different applications

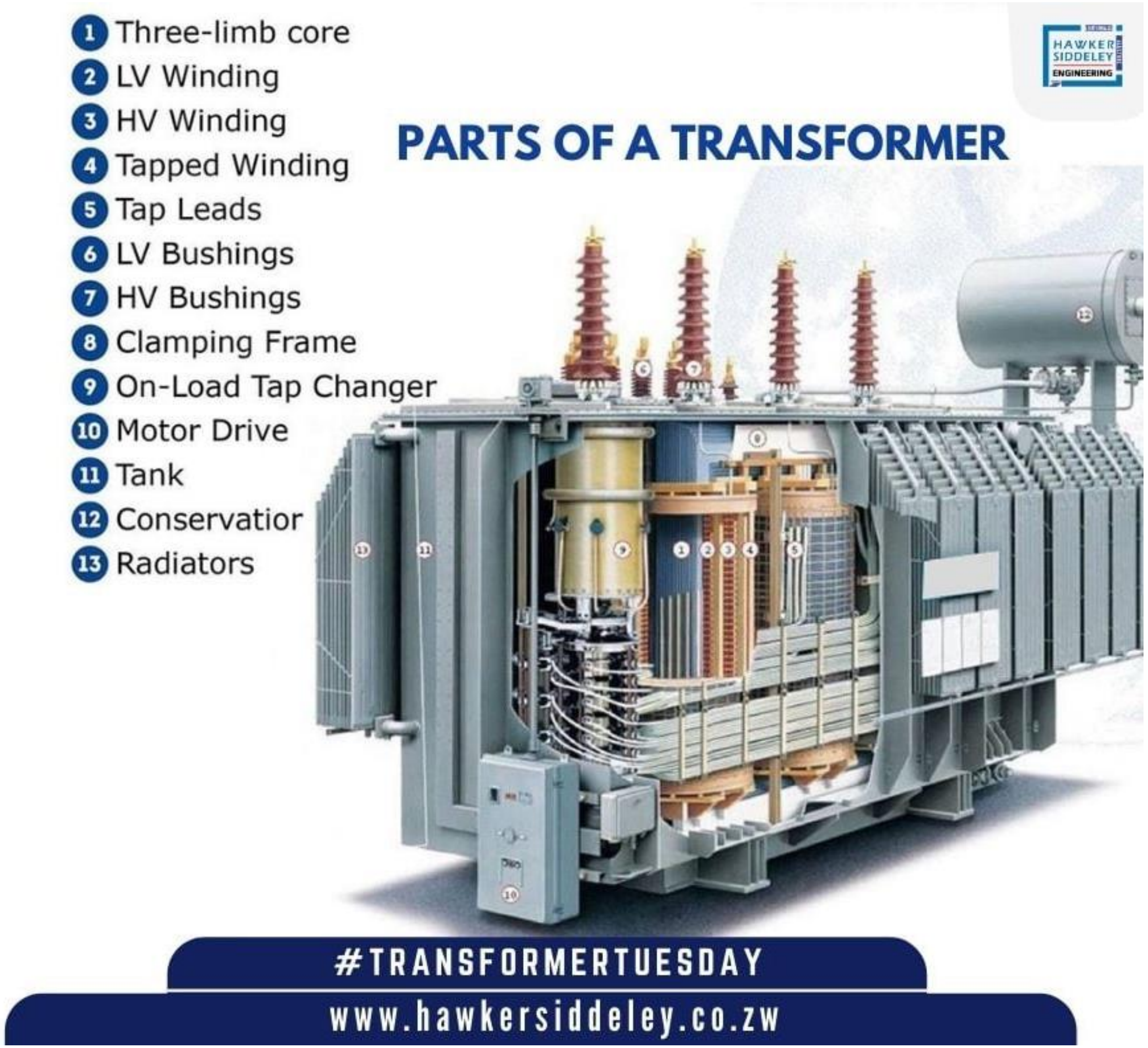
High power transformer

Power transformer

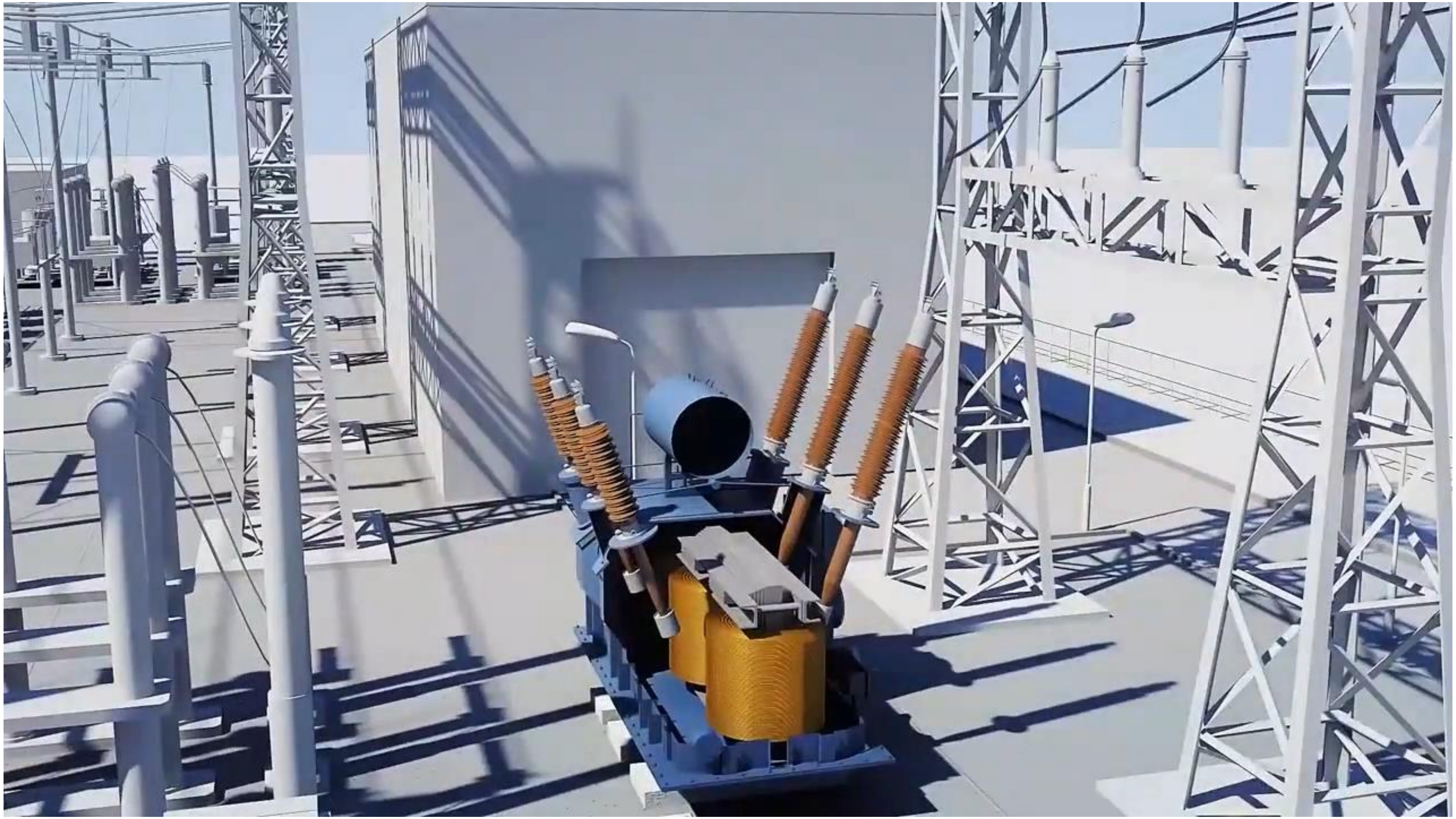
High frequency transformer



Power transformers



Transformer bushings How it work



https://www.youtube.com/watch?v=R2KLxQrVFjU&ab_channel=EEEBElectricalEngineersExperienceBuilding

Example: Ideal single-phase transformer

A single-phase two-winding transformer is rated 20 kVA, 480/120 V, 60 Hz. A source connected to the 480-V winding supplies an impedance load connected to the 120-V winding. The load absorbs 15 kVA at 0.8 p.f. lagging when the load voltage is 118 V. Assume that the transformer is ideal and calculate the following:

a. The voltage across the 480-V winding.
b. The load impedance.
c. The load impedance referred to the 480-V winding.
d. The real and reactive power supplied to the 480-V winding.

SOLUTION

a. Selecting the load voltage $\boldsymbol{E_2}$ as the reference

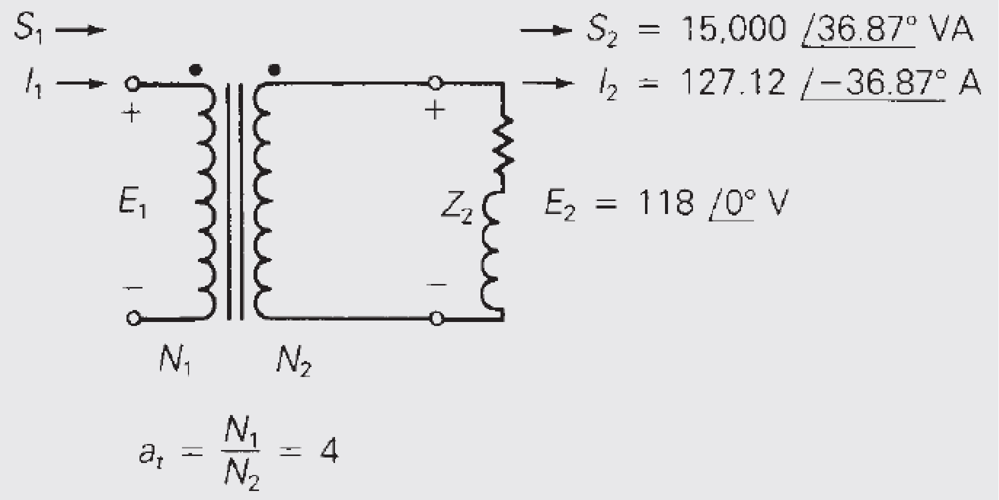
$$E_2 = 118\underline{/0^\circ} \quad \text{V}$$

The turns ratio is, from (3.1.13),

$$a_t = \frac{N_1}{N_2} = \frac{\mathrm{E}_{1\text{rated}}}{\mathrm{E}_{2\text{rated}}} = \frac{480}{120} = 4$$

and the voltage across winding 1 is

$$E_1 = a_t E_2 = 4(118\underline{/0^\circ}) = 472\underline{/0^\circ} \quad \text{V}$$



Example: Ideal single-phase transformer

SOLUTION

b. The complex power S_2 absorbed by the load is

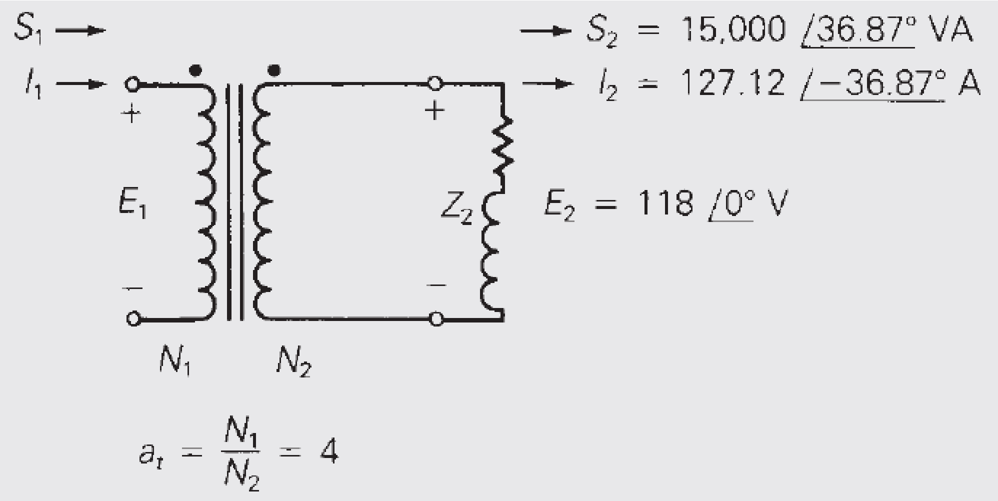
$$S_2 = E_2 I_2^* = 118 I_2^* = 15{,}000\underline{/\cos^{-1}(0.8)} = 15{,}000\underline{/36.87^\circ} \quad \text{VA}$$

Solving, the load current I_2 is

$$I_2 = 127.12\underline{/-36.87^\circ} \quad \text{A}$$

The load impedance Z_2 is

$$Z_2 = \frac{E_2}{I_2} = \frac{118\underline{/0^\circ}}{127.12\underline{/-36.87^\circ}} = 0.9283\underline{/36.87^\circ} \quad \Omega$$



c. The load impedance referred to the 480-V winding is

$$Z_2' = a_t^2 Z_2 = (4)^2(0.9283\underline{/36.87^\circ}) = 14.85\underline{/36.87^\circ} \quad \Omega$$

d. Since $S_1 = S_2 = 15{,}000\underline{/36.87^\circ} = 12{,}000 + j9000$

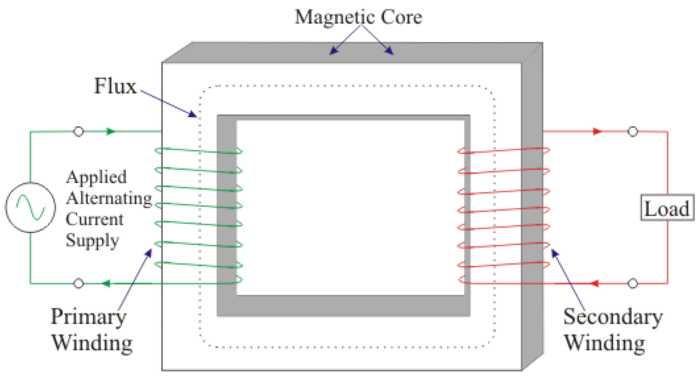
Thus, the real and reactive powers supplied to the 480-V winding are

$$P_1 = \text{Re } S_1 = 12{,}000 \text{ W} = 12 \text{ kW}$$

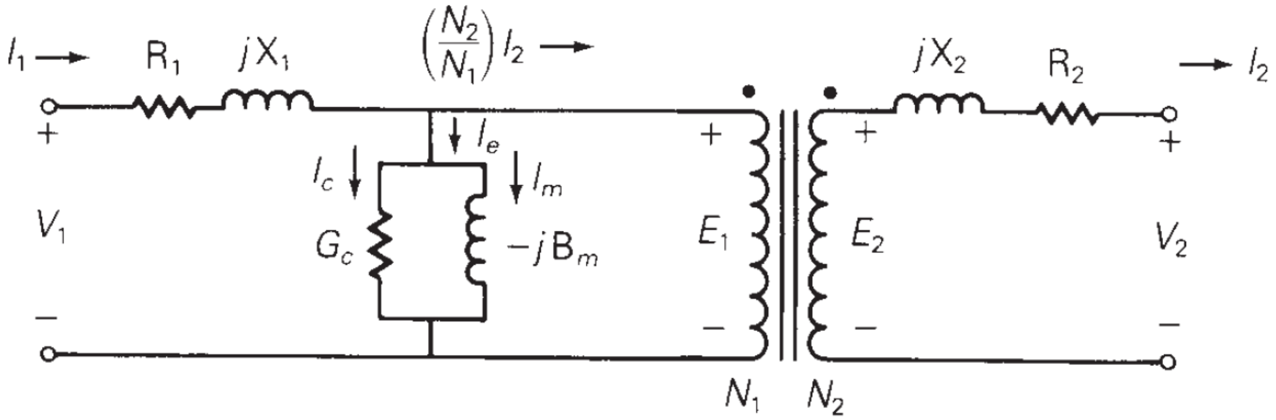
$$Q_1 = \text{Im } S_1 = 9000 \text{ var} = 9 \text{ kvar}$$

Equivalent circuits for practical transformers

Single-phase two winding-transformer



Single Phase Transformer



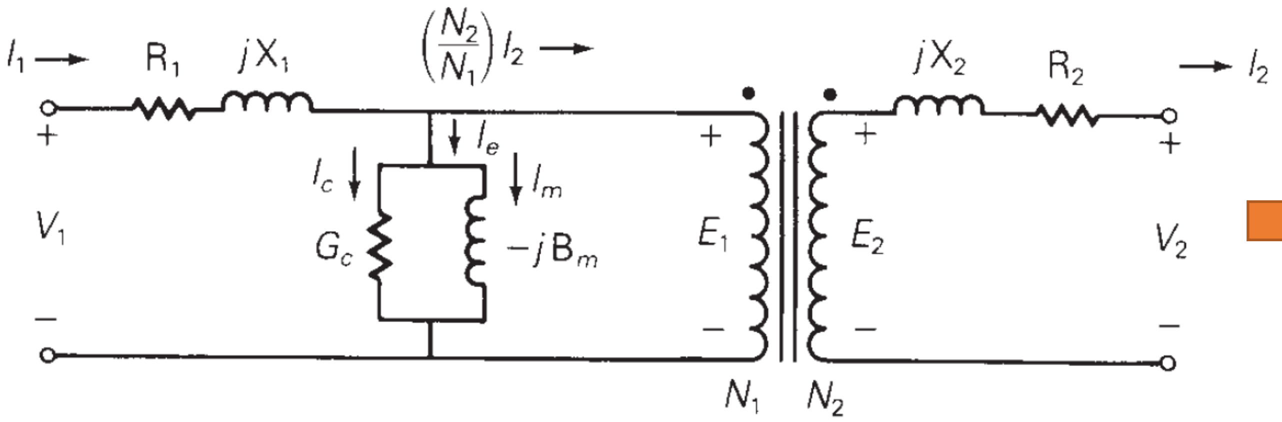
- The resistance R_1 is included in series with winding 1 of the figure to account for I^2R losses in this winding.
- A reactance X_1, called the leakage reactance of winding 1, is also included in series with winding 1 to account for the leakage flux of winding 1. This leakage flux is the component of the flux that links winding 1 but does not link winding 2; it causes a voltage drop $I_1(jX_1)$, which is proportional to I_1 and leads I_1 by 90.
- There is also a reactive power loss $I_1^2X_1$ associated with this leakage reactance.
- Similarly, there is a resistance R_2 and a leakage reactance X_2 in series with winding 2.
- I_m, called magnetizing current, it is evident that I_m lags E_1 by 90
- I_c, called the core loss current, it is in phase with E_1

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = I_c + I_m = (\mathrm{G}_c - j\mathrm{B}_m)E_1$$

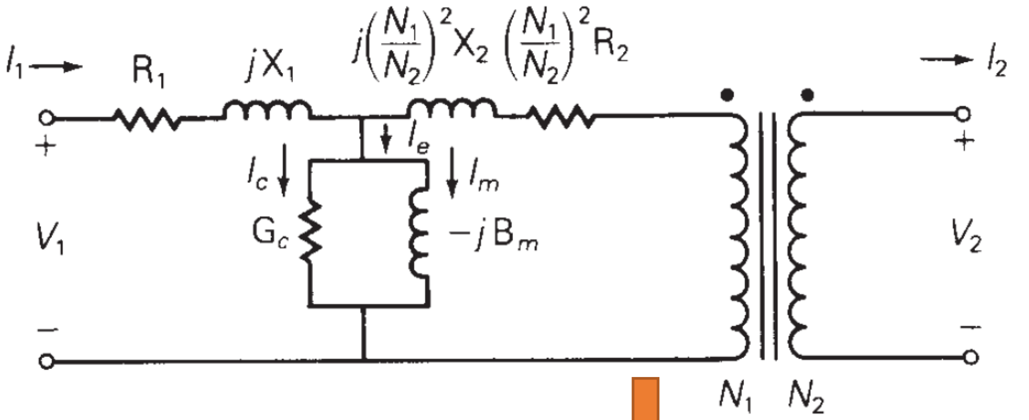
$(\mathrm{G}_c - j\mathrm{B}_m)$ is the admittance of the shunt branch.

Equivalent circuits for practical transformers

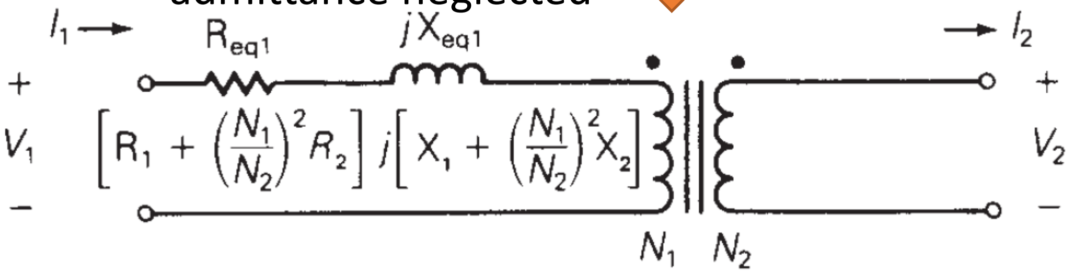
Single-phase two winding-transformer



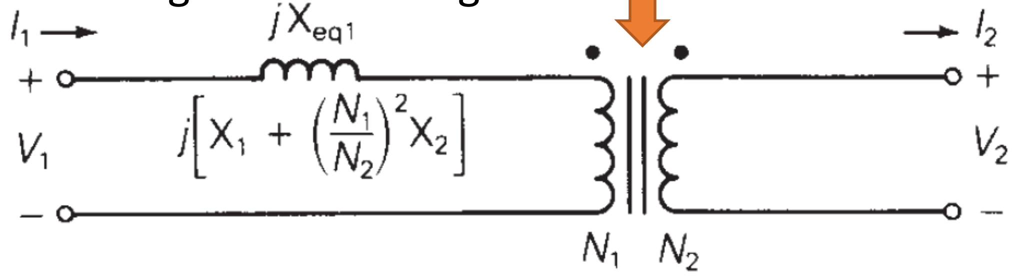
R$_2$ and X$_2$ are referred to winding 1



admittance neglected



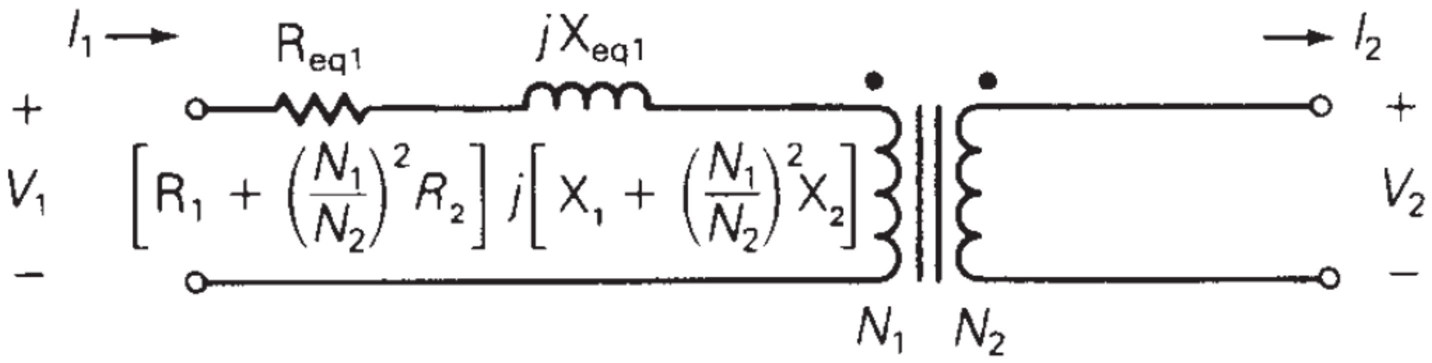
winding resistance neglected



- The above indicates that the current I_e will have two components: the core loss current I_c and the magnetizing current I_m. Associated with I_c is a real power loss $I^2_c/G_c = E^2{}_1 G_c$ W.
- Associated with I_m is a reactive power loss $I^2_m/B_m = E^2{}_1 B_m$ var. This reactive power is required to magnetize the core. The phasor sum $(I_c + I_m)$ is called the exciting current I_e.
- This real power loss, caused by I_c, accounts for both hysteresis and eddy current losses within the core.
- Generally, the admittance can be neglected during calculations.

Equivalent circuits for practical transformers

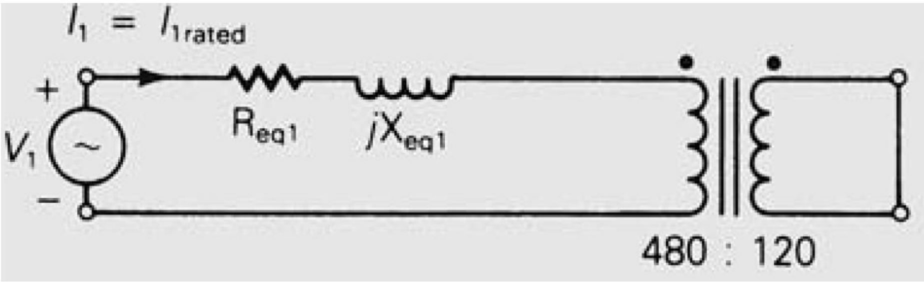
admittance neglected



- Since the exciting current is usually less than 5% of rated current, neglecting it in power system studies is often valid unless transformer efficiency or exciting current phenomena are of particular concern.
- Thus, a practical transformer operating in sinusoidal steady state is equivalent to an ideal transformer with external impedance.
- The external impedance can be evaluated from short-circuit test.

Example: transformer short-circuit

A single-phase two-winding transformer is rated 20 kVA, 480/120 volts, 60 Hz. During a short-circuit test, where rated current at rated frequency is applied to the 480-volt winding (denoted winding 1), with the 120-volt winding (winding 2) shorted, the following readings are obtained: V1 = 35 volts, P1 = 300 W.



admittance neglected

- From the short-circuit test, determine the equivalent series impedance

SOLUTION

The equivalent circuit for the short-circuit test is shown, where the shunt admittance branch is neglected.

Rated current for winding 1 is

$$I_{1rated} = \frac{S_{rated}}{V_{1rated}} = \frac{20 \times 10^3}{480} = 41.667 \quad A$$

R_{eq1}, Z_{eq1}, and X_{eq1} are then determined as follows:

$$R_{eq1} = \frac{P_1}{I^2_{1rated}} = \frac{300}{(41.667)^2} = 0.1728 \quad \Omega$$

$$|Z_{eq1}| = \frac{V_1}{I_{1rated}} = \frac{35}{41.667} = 0.8400 \quad \Omega$$

$$X_{eq1} = \sqrt{Z^2_{eq1} - R^2_{eq1}} = 0.8220 \quad \Omega$$

$$Z_{eq1} = R_{eq1} + jX_{eq1} = 0.1728 + j0.8220 = 0.8400\underline{/78.13^\circ} \quad \Omega$$

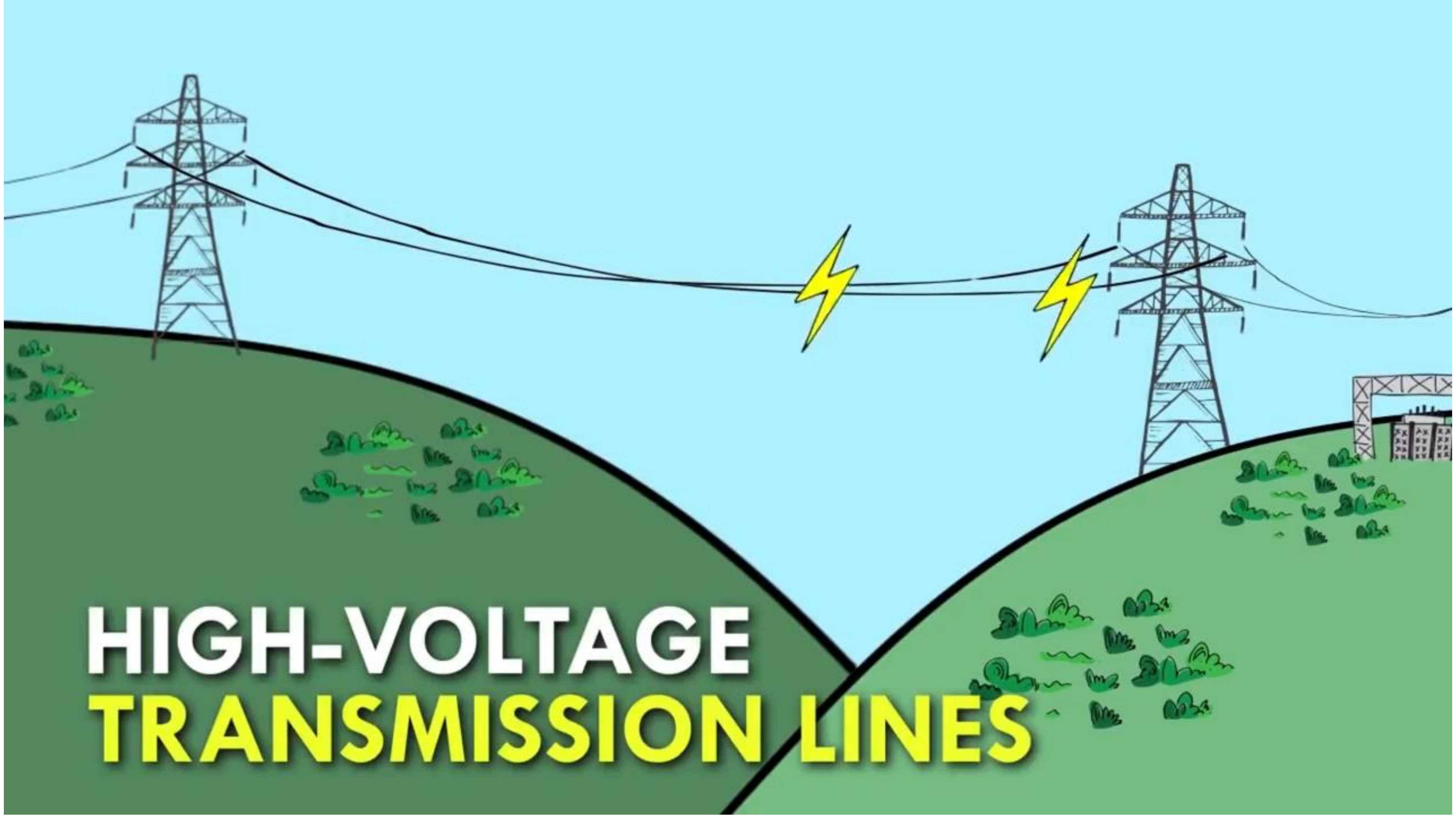
Transmission lines

Transmission lines

Most transmission lines are **high-voltage three-phase alternating current (AC)**, although single phase AC is sometimes used in railway electrification systems. High-voltage direct-current (HVDC) technology is used for greater efficiency over very long distances (typically hundreds of km).

In Hong Kong, CLP transmits power at 400 kV and 132 kV AC, distributes power to customers at 33 kV, 11 kV and 380 V 3-phase. Moreover, HKE transmits at 275 kV and 132 kV AC, and distributes at 22 kV, 11 kV and 380 V. The final home consumers receive power at 220 V single-phase or 380 V three-phase.

Transmission lines: Electric Grid Connects Us All



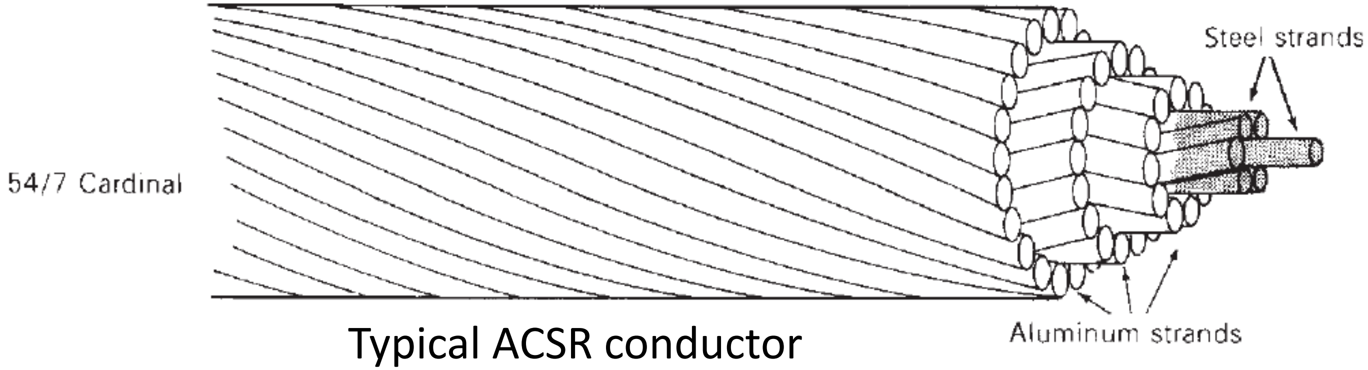
https://www.youtube.com/watch?v=KLS7WGKn4kc&ab_channel=StraightTalkAlert

Transmission line: Conductors

- Aluminum has replaced copper as the most common conductor metal for overhead transmission. Although a larger aluminum cross-sectional area is required to obtain the same loss as in a copper conductor, aluminum has a lower cost and lighter weight. Also, the supply of aluminum is abundant, whereas that of copper is limited.

- One of the most common conductor types is aluminum conductor, steel-reinforced (ACSR), which consists of layers of aluminum strands surrounding a central core of steel strands.

- The use of steel strands gives ACSR conductors a high strength-to-weight ratio. For purposes of heat dissipation, overhead transmission-line conductors are bare (no insulating cover).

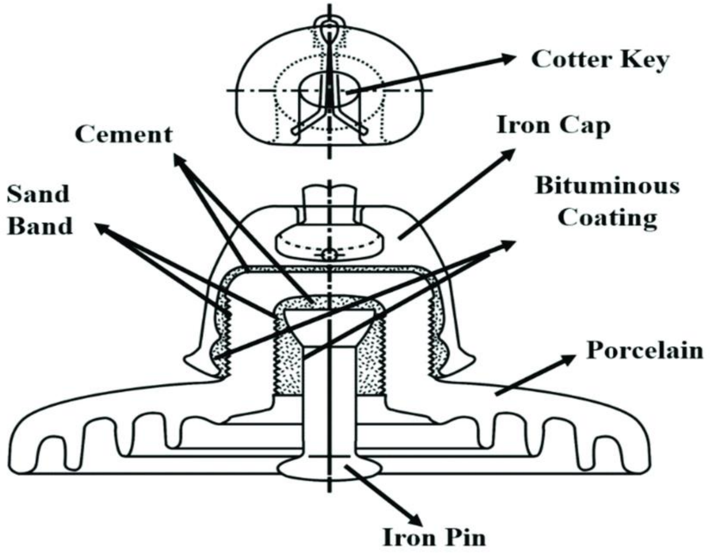


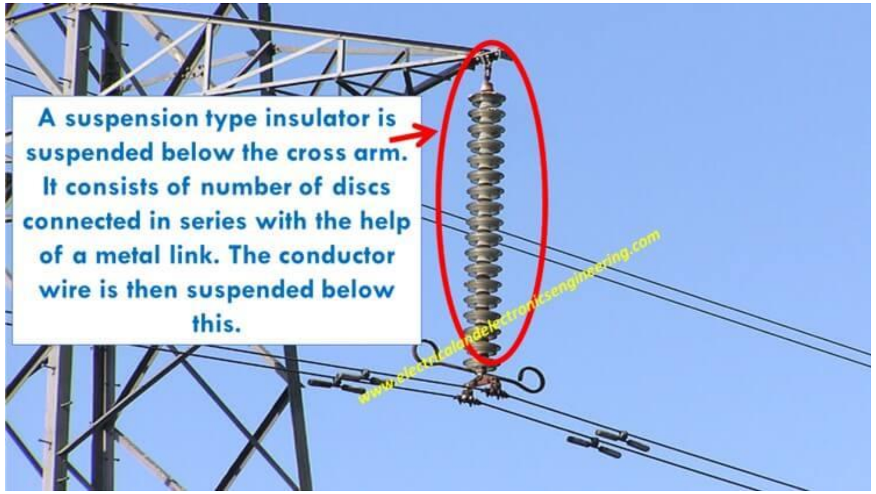
Typical ACSR conductor

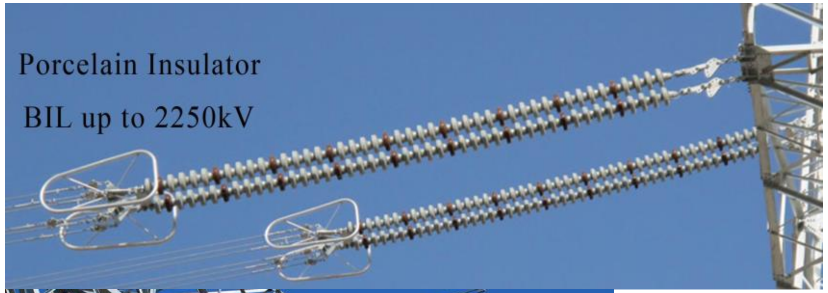
Transmission line: Insulators

- Insulators for transmission lines above 69 kV are typically suspension-type insulators, that consist of a string of discs constructed of porcelain, toughened glass, or polymer.

- The number of insulator discs in a string increases with line voltage. Other types of discs include larger units with higher mechanical strength and fog insulators for use in contaminated areas.

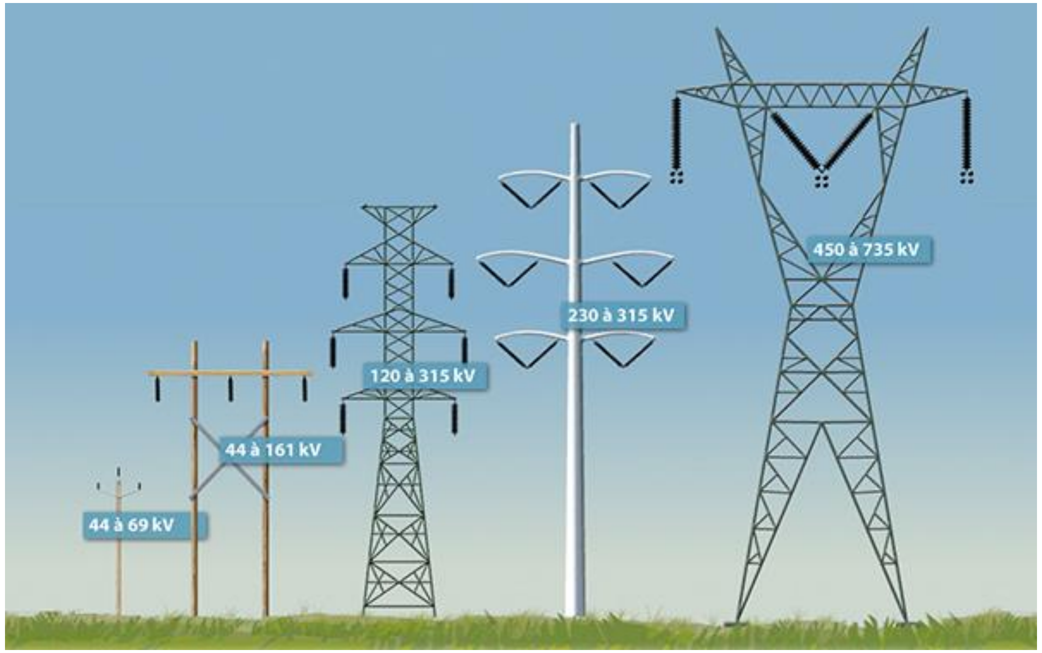


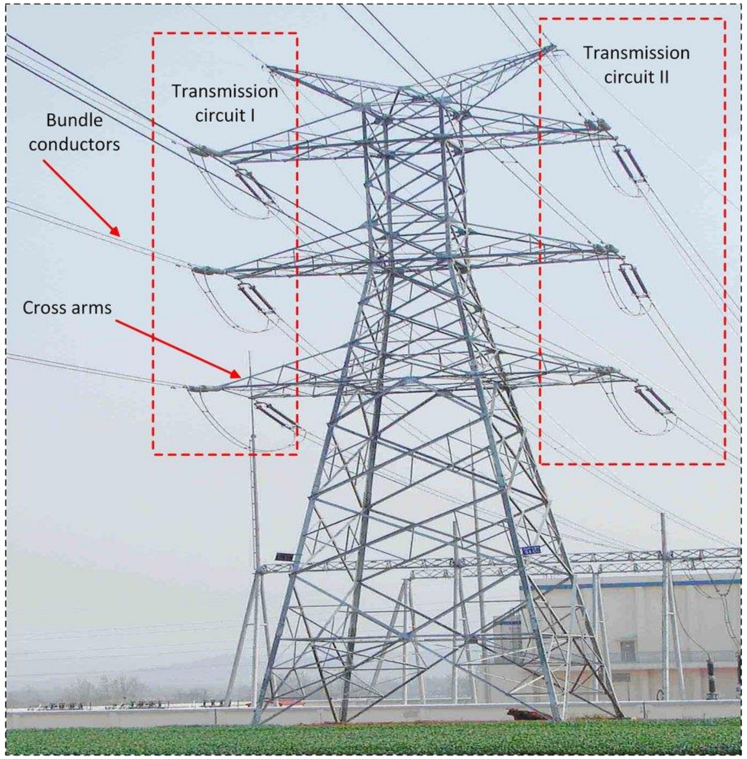




Transmission line: Support structures

- Transmission lines employ a variety of support structures, such as a self-supporting, lattice steel tower typically used for 500- and 765-kV lines.
- Double-circuit 345-kV lines usually have self-supporting steel towers with the phases arranged either in a triangular configuration to reduce tower height or in a vertical configuration to reduce tower width.
- Wood frame configurations are commonly used for voltages of 345 kV and below (not popular for now).

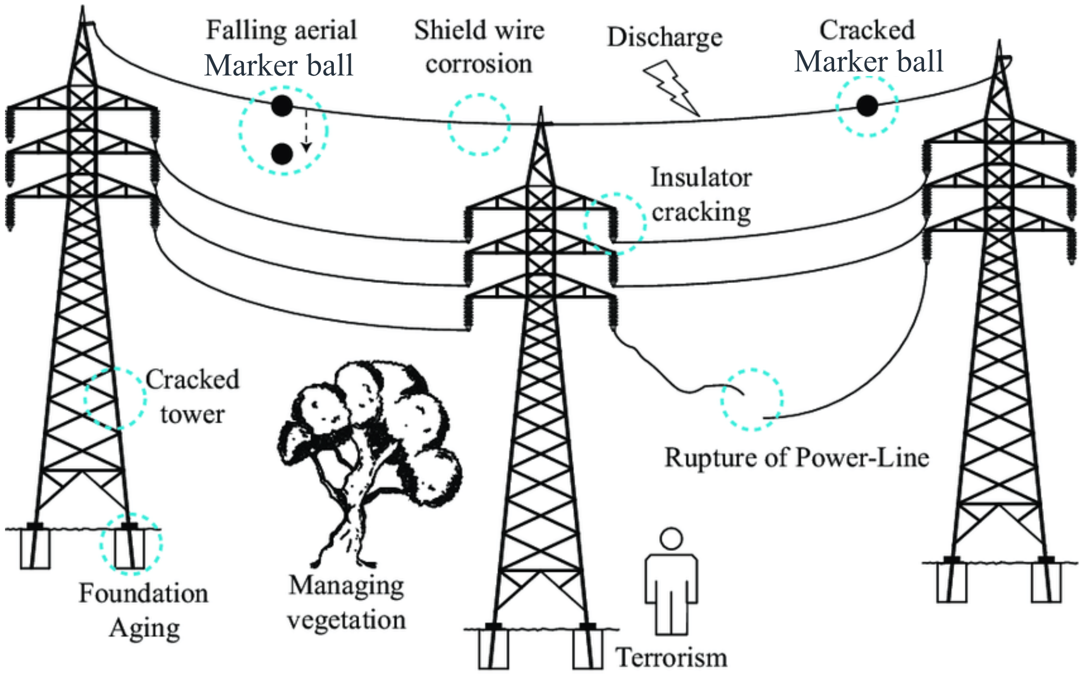




Transmission line: Shield wires

- Shield wires located above the phase conductors protect the phase conductors against lightning. They are usually high- or extra-high-strength steel, Alumoweld, or ACSR with much smaller cross section than the phase conductors.

- Shield wires are grounded to the tower. As such, when lightning strikes a shield wire, it flows harmlessly to ground, provided the tower impedance and tower footing resistance are small.



Typical transmission-line characteristics

- Aluminum has replaced copper as the most common conductor metal for overhead

Nominal Voltage (kV)	Phase Conductors: Number of Conductors per Bundle	Aluminum Cross-Section Area per Conductor (ACSR) (kcmil)	Bundle Spacing (cm)	Minimum Clearances: Phase-to-Phase (m)	Minimum Clearances: Phase-to-Ground (m)
69	1	—	—	—	—
138	1	300–700	—	4 to 5	—
230	1	400–1000	—	6 to 9	—
345	1	2000–2500	—	6 to 9	7.6 to 11
345	2	800–2200	45.7	6 to 9	7.6 to 11
500	2	2000–2500	45.7	9 to 11	9 to 14
500	3	900–1500	45.7	9 to 11	9 to 14
765	4	900–1300	45.7	13.7	12.2

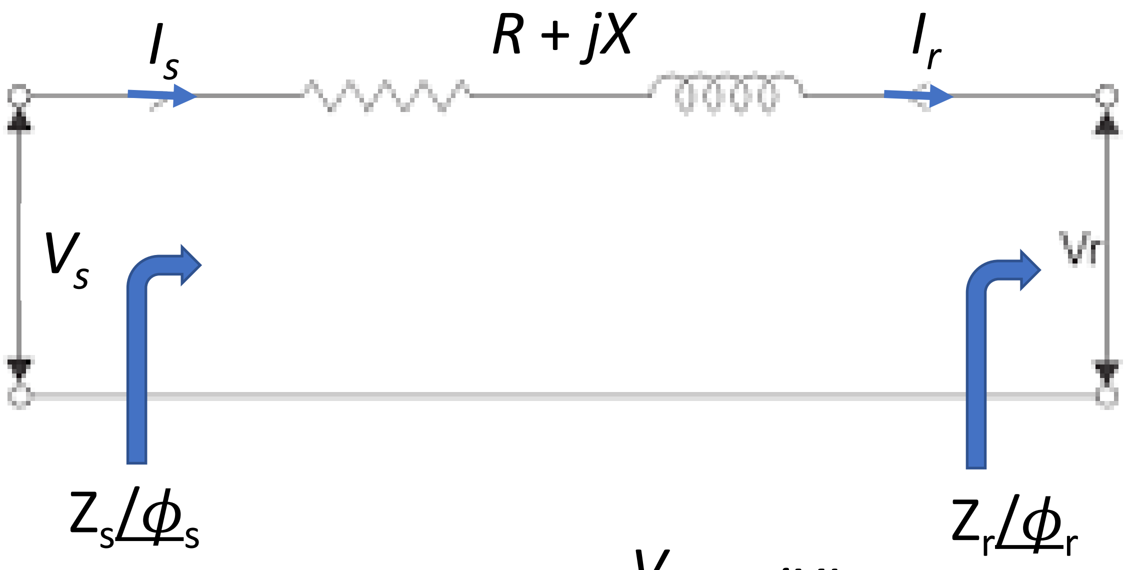
Nominal Voltage (kV)	Suspension Insulator String: Number of Strings per Phase	Suspension Insulator String: Number of Standard Insulator Discs per Suspension String	Shield Wires: Type	Shield Wires: Number	Shield Wires: Diameter (cm)
69	1	4 to 6	Steel	0, 1 or 2	—
138	1	8 to 11	Steel	0, 1 or 2	—
230	1	12 to 21	Steel or ACSR	1 or 2	1.1 to 1.5
345	1	18 to 21	Alumoweld	2	0.87 to 1.5
345	1 and 2	18 to 21	Alumoweld	2	0.87 to 1.5
500	2 and 4	24 to 27	Alumoweld	2	0.98 to 1.5
500	2 and 4	24 to 27	Alumoweld	2	0.98 to 1.5
765	2 and 4	30 to 35	Alumoweld	2	0.98

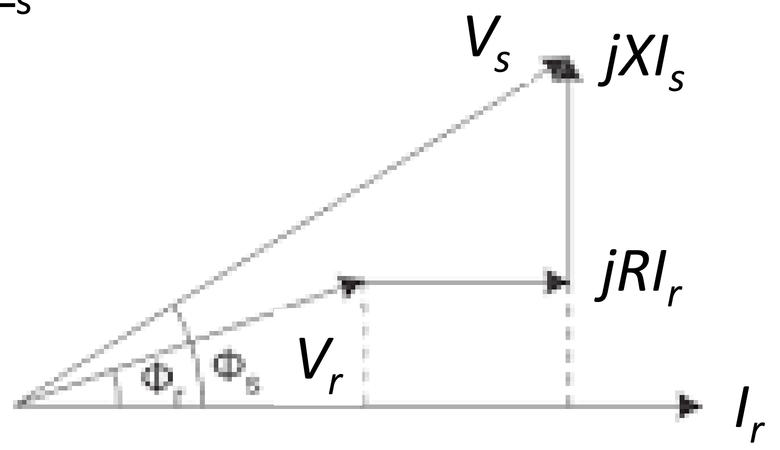
Transmission line model

For short lines less than 50 km, we may assume it is a series RL circuit.

Hence, it causes lagging power factor, i.e., the source side has more leading phase shift than the load side.

Basically, the shunt capacitance is ignored for short lines.



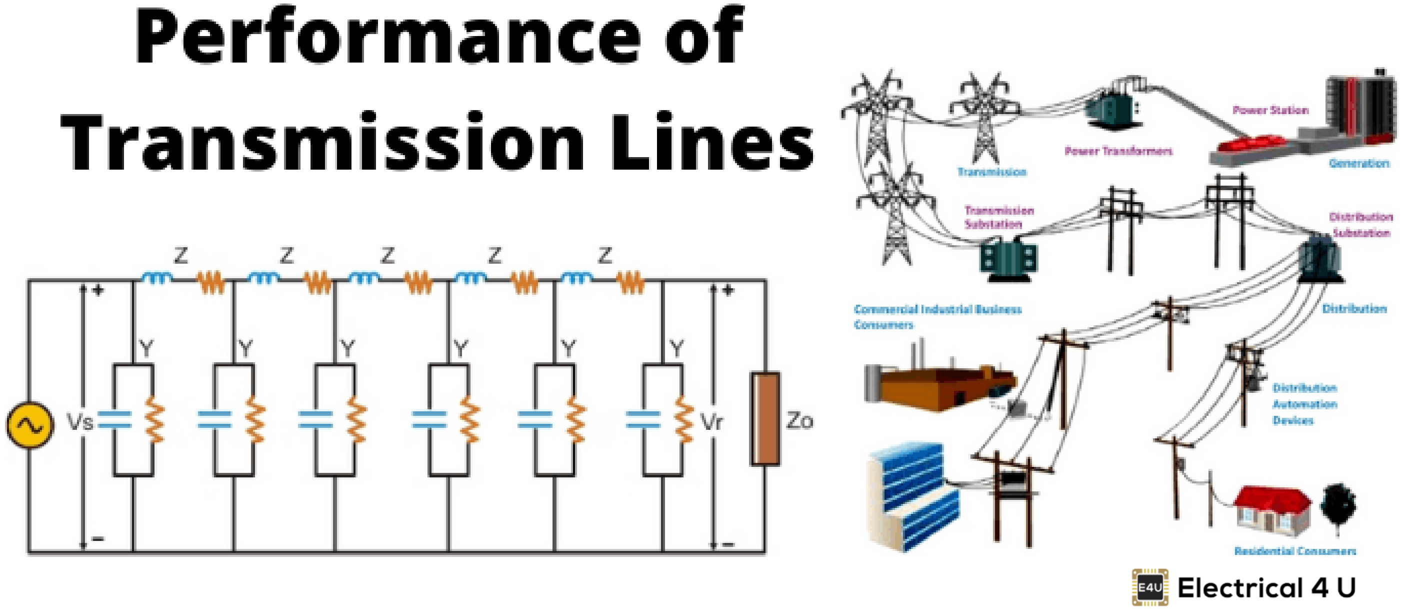


Long distance lines

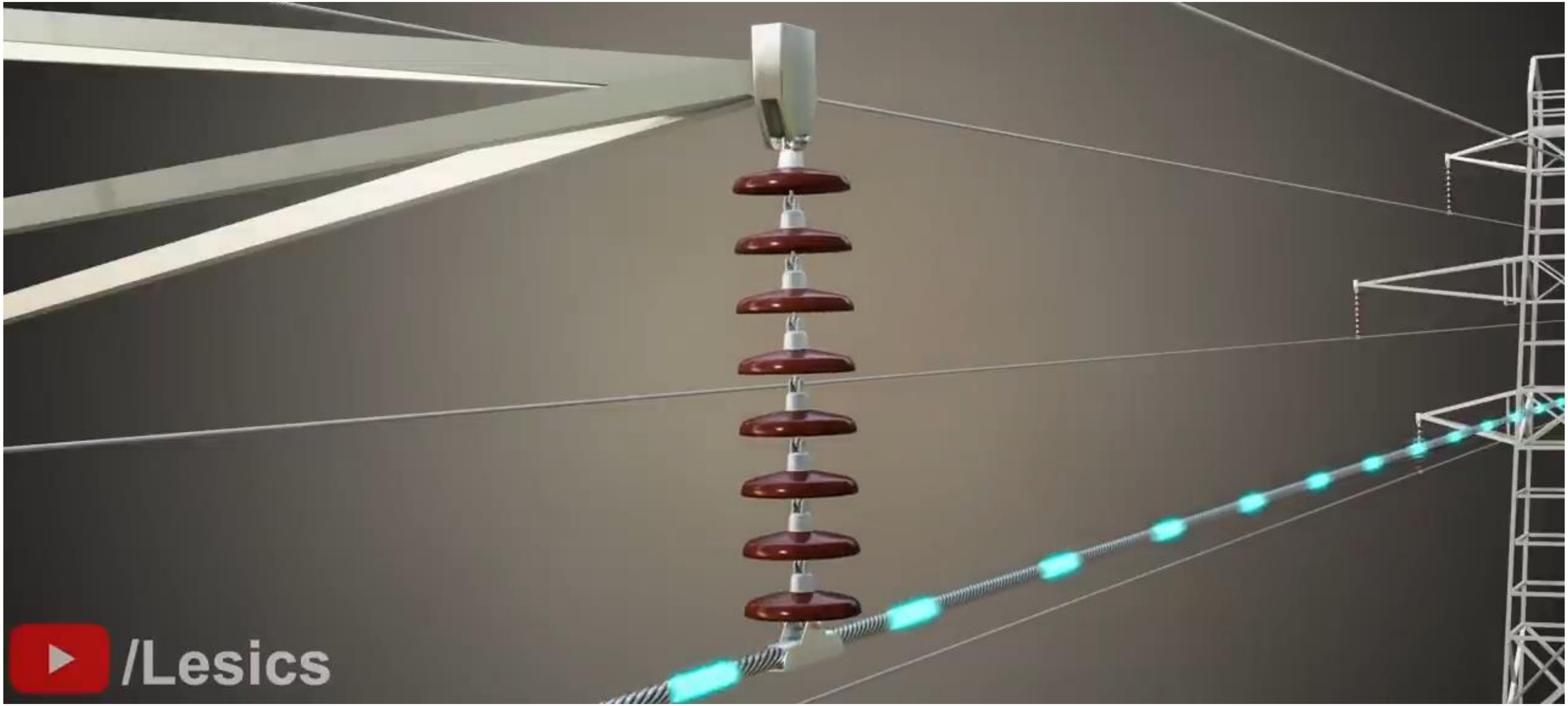
For lines longer than 250 km, we need to take into account the shunt capacitors. For very long lines, we cannot assume lumped circuits and should adopt distributed circuit models, like in communication systems.

Why? What is the wavelength for 50 Hz waves? Around 6000 km. So, a 500 km long line is 1/12 of the wavelength, and voltage and current vary along different locations of the line significantly. We need to use distributed circuit models.

Discussed in an electromagnetics course.



Electric Insulators | Why are they Crucial?



https://www.youtube.com/watch?v=-xDncegzcW8&ab_channel=Lesics

Per-unit system

- One advantage of the per-unit system is that by properly specifying base quantities, the transformer equivalent circuit can be simplified. The ideal transformer winding can be eliminated, such that voltages, currents, and external impedances and admittances expressed in per-unit do not change when they are referred from one side of a transformer to the other.
- Voltage, current, power and impedance can be expressed in per-unit which is the percent of specified base values.

$$\text{per-unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

By convention, we adopt the following two rules for base quantities:

1. The value of $S_{base1\phi}$ is the same for the entire power system of concern.
2. The ratio of the voltage bases on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings.

With these two rules, a per-unit impedance remains unchanged when referred from one side of a transformer to the other.

Per-unit system

$$\text{per-unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Example: Single-phase transformer rated at **20 kVA, 480/120 V and 60 Hz.**
Leakage impedance referred to the secondary side is $0.0525\ \underline{/78.12^o}\ \Omega$.

Base value of secondary impedance = $V^2/S = 120^2/20000 = 0.72\ \Omega$
So, the per-unit leakage at secondary = $(0.0525/0.72)\ \underline{/78.12^o}\ \Omega$ **= 0.0729 $\underline{/78.12^o}$ p.u.**

Base value of primary impedance = $V^2/S = 480^2/20000 = 11.52\ \Omega$
The leakage referred to primary = $(480/120)^2 \times 0.0525\ \underline{/78.12^o} = 0.84\ \underline{/78.12^o}\ \Omega$
So, the per-unit leakage at primary = $(0.84/11.52)\ \underline{/78.12^o}\ \Omega$ **= 0.0729 $\underline{/78.12^o}$ p.u.**

SAME

Per-unit conversion for impedance

- When only one component, such as a transformer, is considered, the nameplate ratings of that component are usually selected as base values.
- When several components are involved, however, the system base values may be different from the nameplate ratings of any particular device.
- It is then necessary to convert the per-unit impedance of a device from its nameplate ratings to the system base values. To convert a per-unit impedance from ''old'' to ''new'' base values, use

$$Z_{\text{pu,new}} = \frac{Z_{\text{actual}}}{Z_{\text{basenew}}} = \frac{Z_{\text{pu,old}} Z_{\text{baseold}}}{Z_{\text{basenew}}}$$

or,

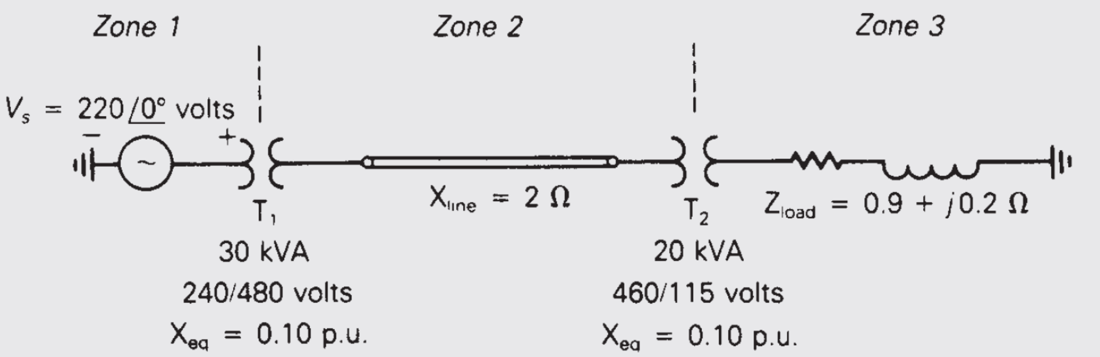
$$Z_{\text{pu,new}} = Z_{\text{pu,old}} \left(\frac{V_{\text{baseold}}^2 / S_{\text{baseold}}}{V_{\text{basenew}}^2 / S_{\text{basenew}}} \right)$$

Example: three-zone single-phase network

Three zones of a single-phase circuit are identified below. The zones are connected by transformers T_1 and T_2, whose ratings are also shown. Using base values of 30 kVA and 240 volts in zone 1,

- draw the per-unit circuit and determine the per-unit impedances and the per-unit source voltage.
- Then calculate the load current both in per-unit and in amperes.

Transformer winding resistances and shunt admittance branches are neglected.



SOLUTION

First the base values in each zone are determined. S_{base} = 30 kVA is the same for the entire network. Also, V_{base1} =240 volts, as specified for zone 1. When moving across a transformer, the voltage base is changed in proportion to the transformer voltage ratings. Thus,

$$V_{base2} = \left(\frac{480}{240}\right)(240) = 480 \quad \text{volts}$$

and

$$V_{base3} = \left(\frac{115}{460}\right)(480) = 120 \quad \text{volts}$$

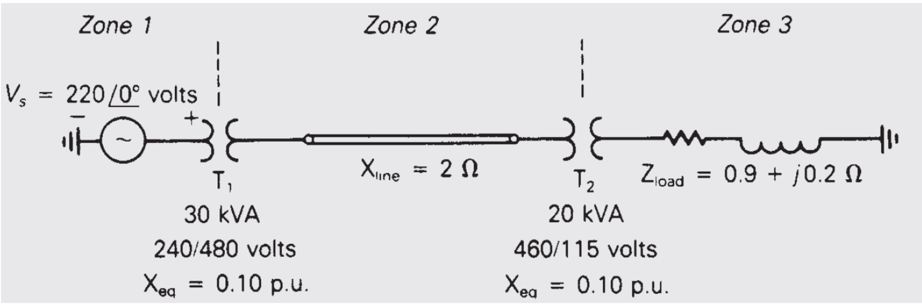
$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{480^2}{30{,}000} = 7.68 \quad \Omega$$

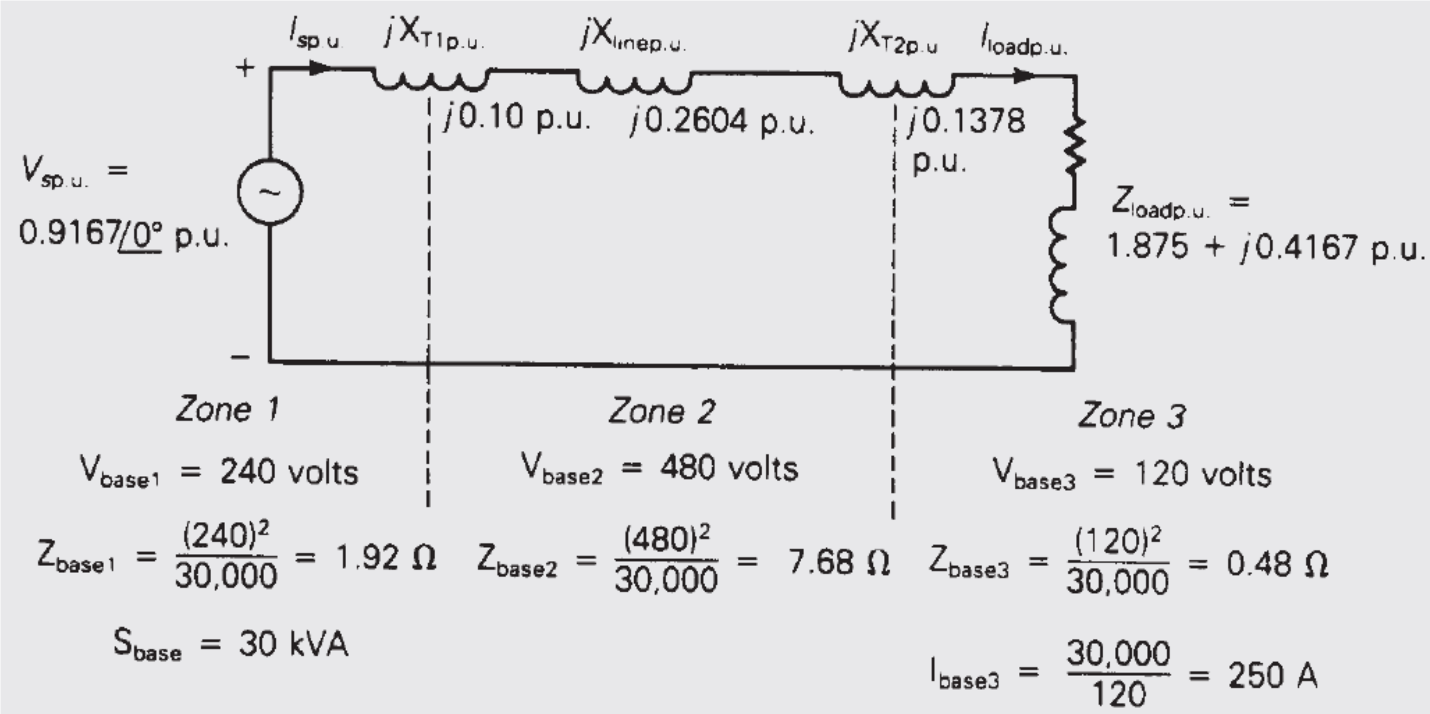
and

$$Z_{base3} = \frac{V_{base3}^2}{S_{base}} = \frac{120^2}{30{,}000} = 0.48 \quad \Omega$$

$$I_{base3} = \frac{S_{base}}{V_{base3}} = \frac{30{,}000}{120} = 250 \quad \text{A}$$

Example: three-zone single-phase network





Next, the per-unit circuit impedances are calculated using the system base values. Since S_{base} = 30 kVA is the same as the kVA rating of transformer T_1, and V_{base1} = 240 volts is the same as the voltage rating of the zone 1 side of transformer T_1, the per-unit leakage reactance of T_1 is the same as its nameplate value, $X_{T1p.u.}$ = 0.1 per unit. However, the per-unit leakage reactance of transformer T_2 must be converted from its nameplate rating to the system base. Using new base from the old base, and V_{base2} = 480 volts,

$$X_{T2p.u.} = (0.10)\left(\frac{460}{480}\right)^2\left(\frac{30,000}{20,000}\right) = 0.1378 \quad \text{per unit}$$

Alternatively, using V_{base3} = 120 volts,

$$X_{T2p.u.} = (0.10)\left(\frac{115}{120}\right)^2\left(\frac{30,000}{20,000}\right) = 0.1378 \quad \text{per unit}$$

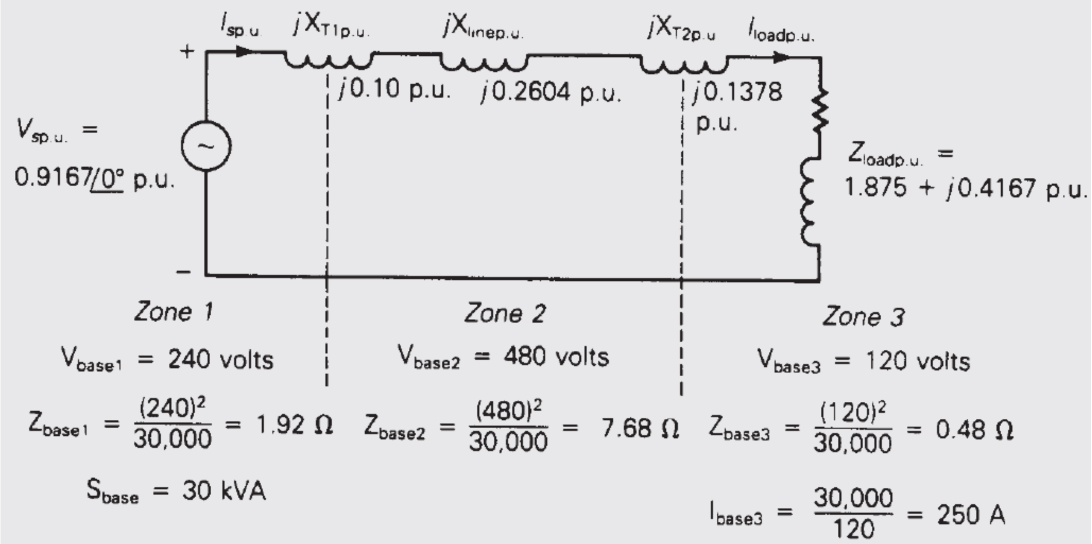
The line, which is located in zone 2, has a per unit reactance

$$X_{linep.u.} = \frac{X_{line}}{Z_{base2}} = \frac{2}{7.68} = 0.2604 \quad \text{per unit}$$

and the load, which is located in zone 3, has a per-unit impedance

$$Z_{loadp.u.} = \frac{Z_{load}}{Z_{base3}} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167 \quad \text{per unit}$$

Example: three-zone single-phase network



The per-unit load current is then easily calculated from

$$
\begin{aligned}
I_{loadp.u.} = I_{sp.u.} &= \frac{V_{sp.u.}}{j(X_{T1p.u.} + X_{linep.u.} + X_{T2p.u.}) + Z_{loadp.u.}} \\
&= \frac{0.9167\underline{/0°}}{j(0.10 + 0.2604 + 0.1378) + (1.875 + j0.4167)} \\
&= \frac{0.9167\underline{/0°}}{1.875 + j0.9149} = \frac{0.9167\underline{/0°}}{2.086\underline{/26.01°}} \\
&= 0.4395\underline{/-26.01°} \quad \text{per unit}
\end{aligned}
$$

The actual load current is

$$I_{load} = (I_{loadp.u.})I_{base3} = (0.4395\underline{/-26.01°})(250) = 109.9\underline{/-26.01°} \quad \text{A}$$

Summary

- Basic ac power circuits
  - Phasor diagrams
  - Power concepts
  - Power factor
- Transformers
  - Coupled inductors
  - Models
- Transmission lines
  - Short line models
  - Long line distributed models
- Per unit system