EE3123 Introduction to Electric Power Systems

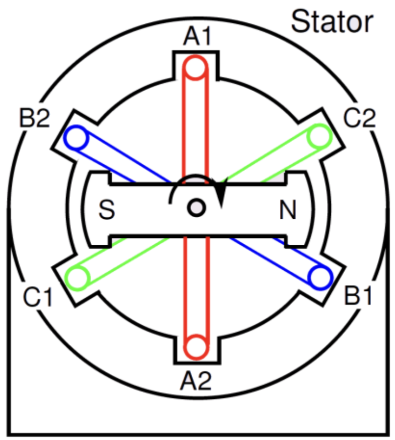
Three-phase power circuit analysis

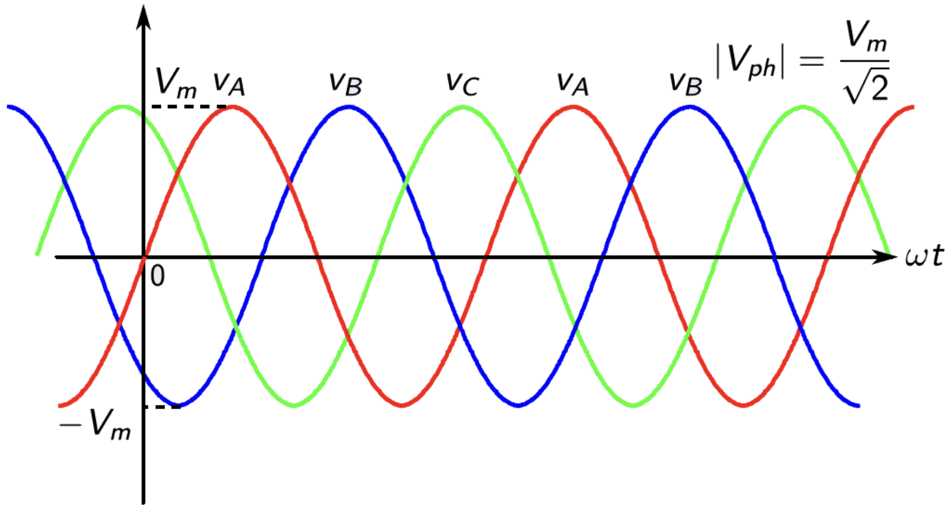
Prof. CQ Jiang

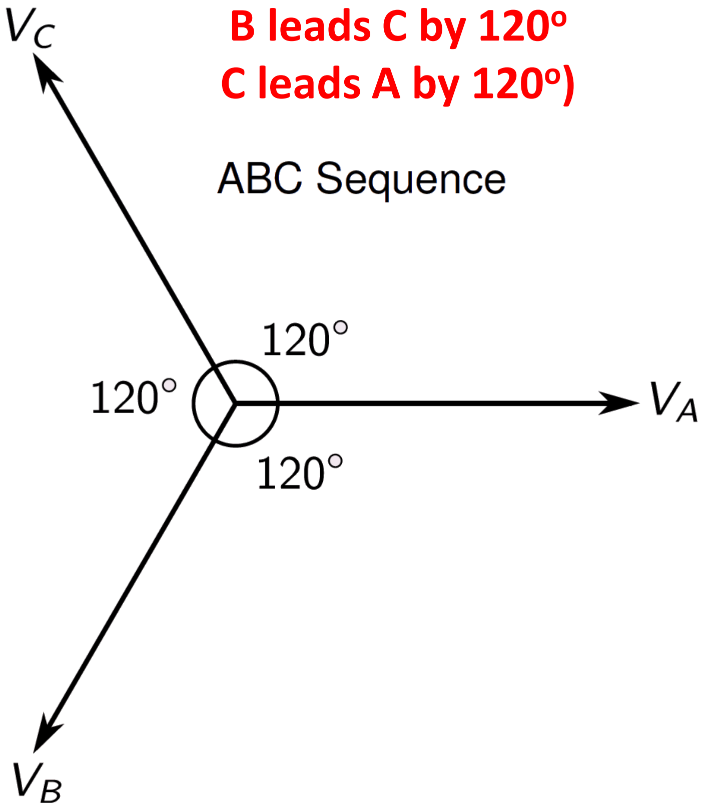
Many thanks to Prof. Michael Tse

Three-phase voltages

Positive sequence
(phasor rotating anticlockwise
A leads B by 120º
B leads C by 120º
C leads A by 120º)



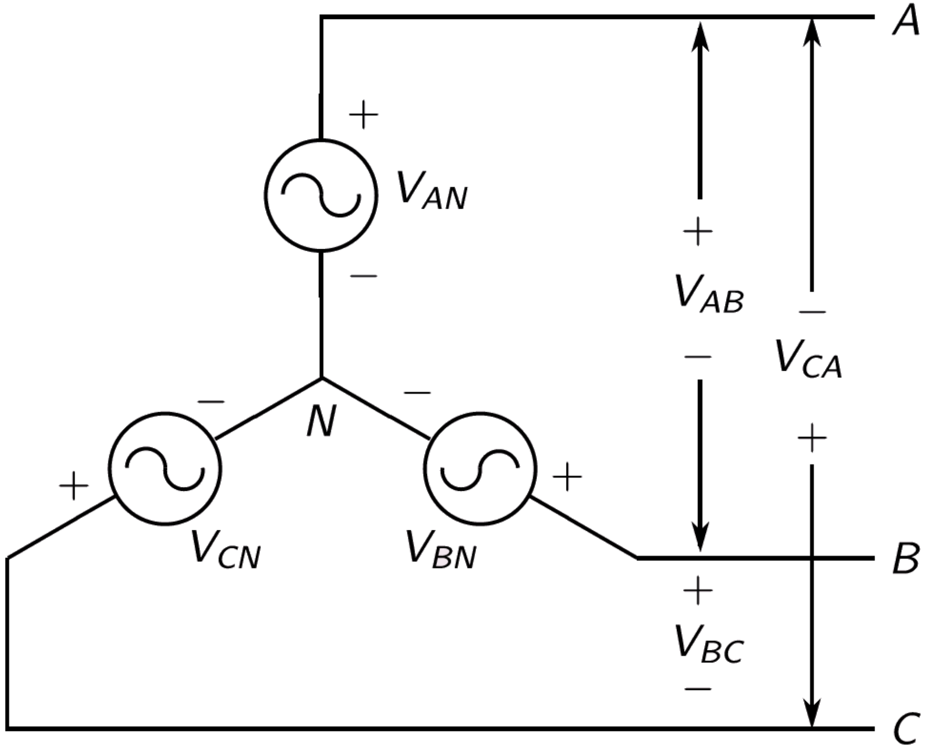




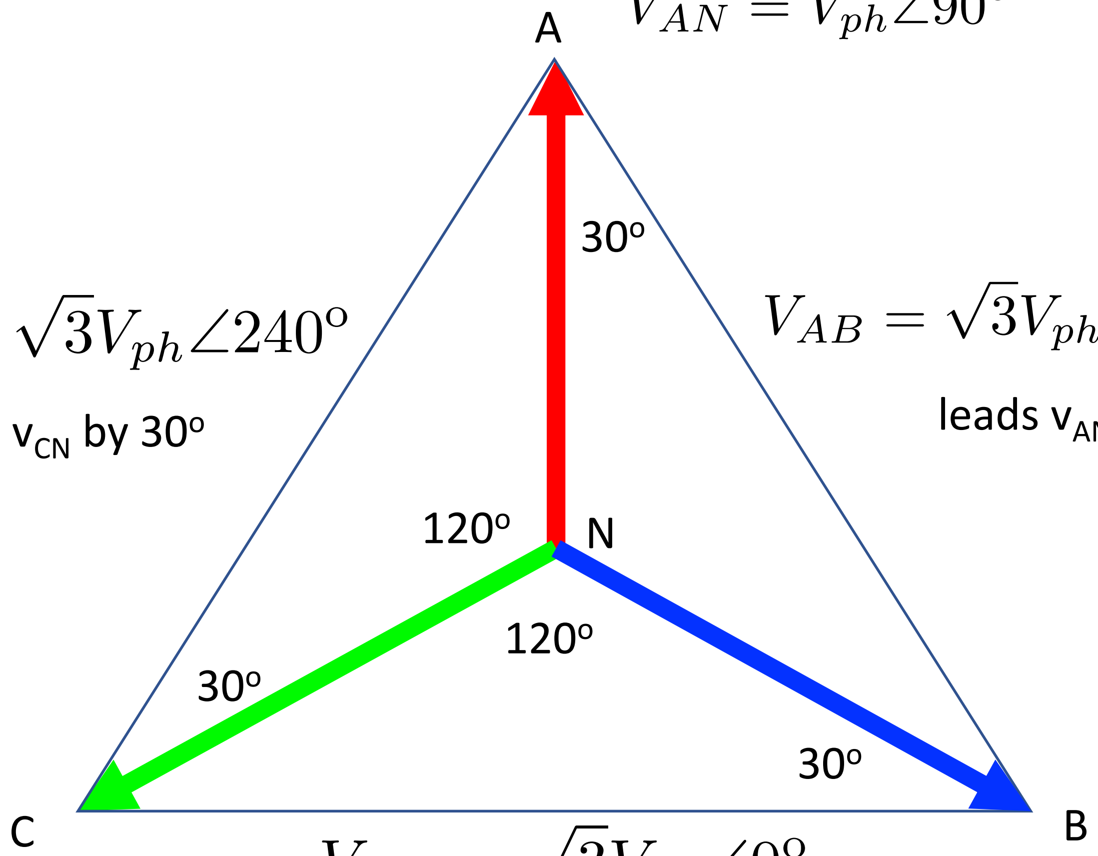
$$v_A = V_m \sin \omega t \Rightarrow V_A = |V_{ph}|\angle 0^{\text{o}}$$
$$v_B = V_m \sin(\omega t - 120^{\text{o}}) \Rightarrow V_B = |V_{ph}|\angle{-120^{\text{o}}}$$
$$v_C = V_m \sin(\omega t - 240^{\text{o}}) \Rightarrow V_C = |V_{ph}|\angle{-240^{\text{o}}}$$

If $|V_A| = |V_B| = |V_C| = |V_{ph}|$ and all phase differences are equal, the three-phase voltages are BALANCED.

POSITIVE SEQUENCE or ABC SEQUENCE



$$E_{ab} = \sqrt{3}E_{an}\underline{/+30^\circ}$$
$$E_{bc} = \sqrt{3}E_{bn}\underline{/+30^\circ}$$
$$E_{ca} = \sqrt{3}E_{cn}\underline{/+30^\circ}$$



$V_{AN} = V_{ph}\angle 90^o$

$V_{CA} = \sqrt{3}V_{ph}\angle 240^o$

leads v_{CN} by 30°

$V_{AB} = \sqrt{3}V_{ph}\angle 120^o$

leads v_{AN} by 30°

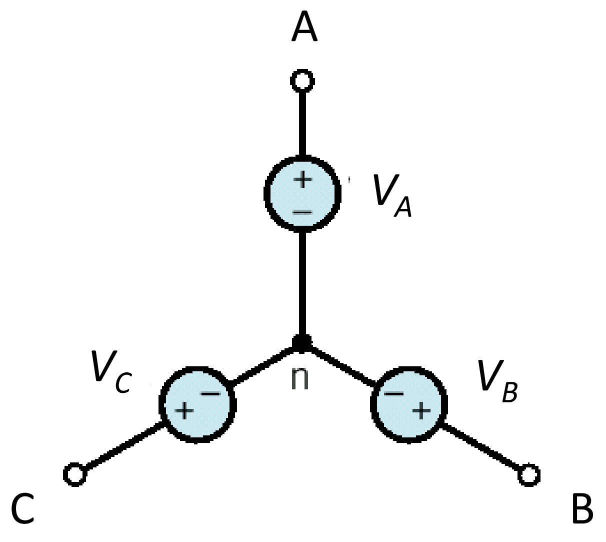
$V_{BC} = \sqrt{3}V_{ph}\angle 0^o$

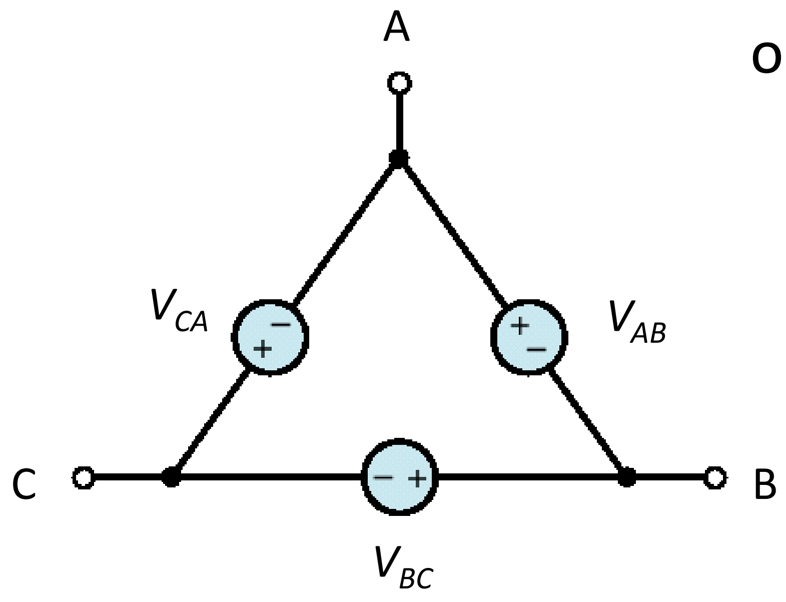
leads v_{BN} by 30°

$V_{CN} = V_{ph}\angle 210^o$

$V_{BN} = V_{ph}\angle -30^o$

Three-phase connections





$$V_{\mathrm{line}} = \sqrt{3}V_{\mathrm{phase}}$$

or

$$V_{\mathrm{LL}} = \sqrt{3}V_{\mathrm{LN}}$$

V_A = 110 V

V_A = 220 V

V_A = 240 V

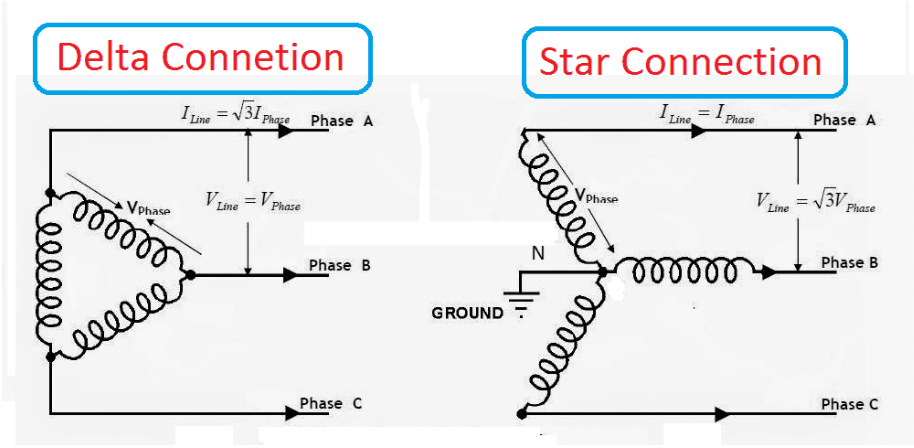
Phase voltage

V_{AC} = 190 V

V_{AC} = 380 V

V_{AC} = 416 V

Line voltage

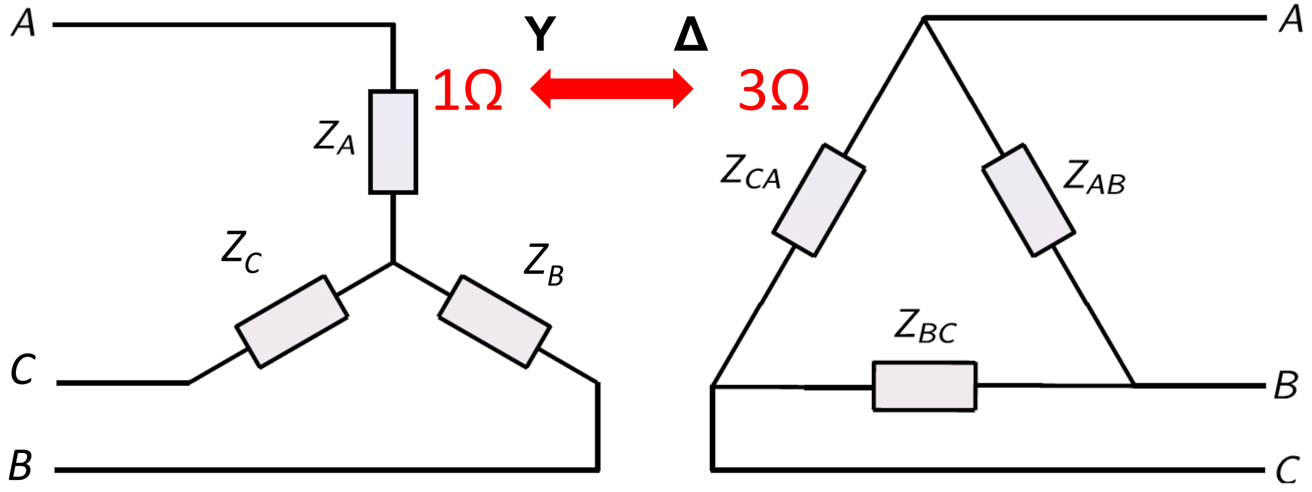


Three-phase balanced loads

If all loads are equal in a Y or Δ circuit, then the three-phase load is BALANCED.

For balanced loads,

$Z_{\Delta} = 3Z_{Y}$



$$Z_A = \frac{Z_{CA}Z_{AB}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_B = \frac{Z_{AB}Z_{AB}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

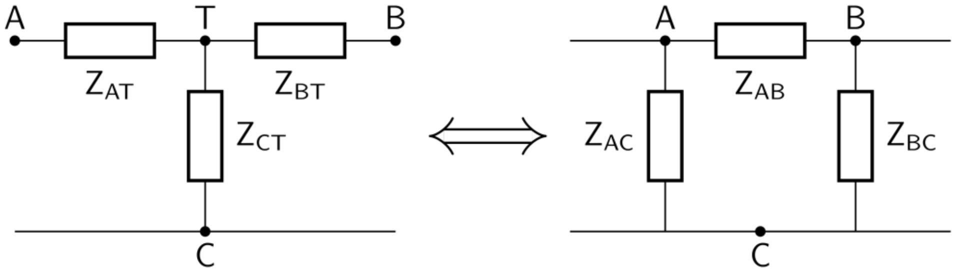
$$Z_C = \frac{Z_{BC}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_{AB} = \frac{Z_AZ_B + Z_BZ_C + Z_CZ_A}{Z_C}$$

$$Z_{BC} = \frac{Z_AZ_B + Z_BZ_C + Z_CZ_A}{Z_A}$$

$$Z_{CA} = \frac{Z_AZ_B + Z_BZ_C + Z_CZ_A}{Z_B}$$

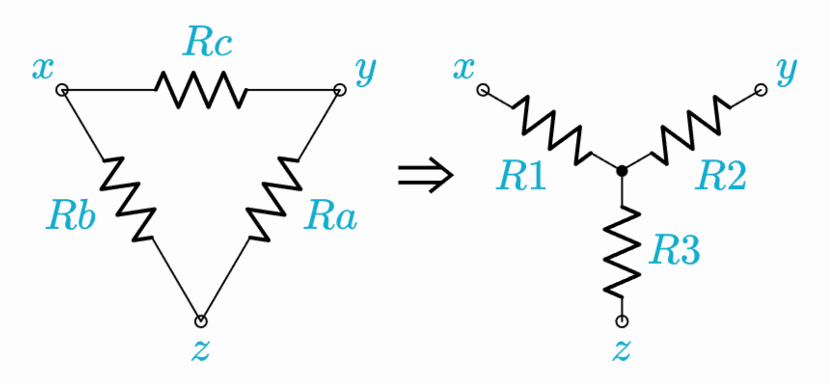
Pi - T transformation



https://www.youtube.com/watch?v=biomymZbK-U&ab_channel=PhysicsNinja

Proof of Wye (Y) – Delta (Δ) Transformation

Δ – Y derivation



In Δ

In Y

$$R_{xy} = Rc \parallel (Ra + Rb) = \frac{Rc\,(Ra + Rb)}{Rc + (Ra + Rb)}$$

$$R_{xy} = R1 + R2$$

$$R_{xy}: \quad R1 + R2 = Rc\,(Ra + Rb)/(Ra + Rb + Rc)$$

The same for other two sub circuits

$$R_{xy}: \quad R1 + R2 = Rc\,(Ra + Rb)/(Ra + Rb + Rc)$$
$$R_{yz}: \quad R2 + R3 = Ra\,(Rb + Rc)/(Ra + Rb + Rc)$$
$$R_{zx}: \quad R3 + R1 = Rb\,(Rc + Ra)/(Ra + Rb + Rc)$$

$$R1 = (\quad R_{xy} \quad + \quad R_{zx} \quad - \quad R_{yz} \quad)/2$$

$$= \frac{Rc(Ra + Rb) + Rb(Rc + Ra) - Ra(Rb + Rc)}{2(Ra + Rb + Rc)}$$

$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc}$$

The same to get R2 and R3.

The Δ to Y transformation is,

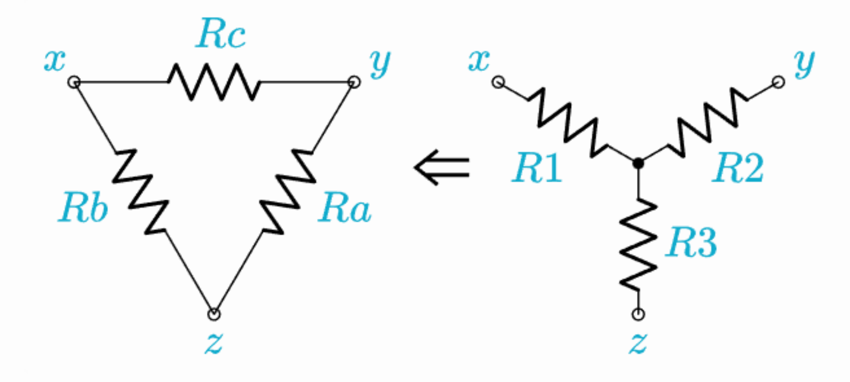
$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc}$$

$$R2 = \frac{Ra\,Rc}{Ra + Rb + Rc}$$

$$R3 = \frac{Ra\,Rb}{Ra + Rb + Rc}$$

Proof of Wye (Y) – Delta (Δ) Transformation

Y – Δ derivation



The Y to Δ transformation is,

$$Ra = \frac{R3\,R2 + R3\,R1 + R1\,R2}{R1}$$

$$Rb = \frac{R3\,R2 + R3\,R1 + R1\,R2}{R2}$$

$$Rc = \frac{R3\,R2 + R3\,R1 + R1\,R2}{R3}$$

The Δ to Y transformation is,

$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc}$$

$$R2 = \frac{Ra\,Rc}{Ra + Rb + Rc}$$

$$R3 = \frac{Ra\,Rb}{Ra + Rb + Rc}$$

$$\frac{R3}{R1} = \frac{\frac{Ra\,Rb}{Ra + Rb + Rc}}{\frac{Rb\,Rc}{Ra + Rb + Rc}}$$

$$\frac{R3}{R1} = \frac{Ra\,Rb}{Rb\,Rc} = \frac{Ra}{Rc}$$

$$Ra = \frac{R3\,Rc}{R1}$$

Repeat the previous

$$\frac{R3}{R2} = \frac{Ra\,Rb}{Ra\,Rc} = \frac{Rb}{Rc}$$

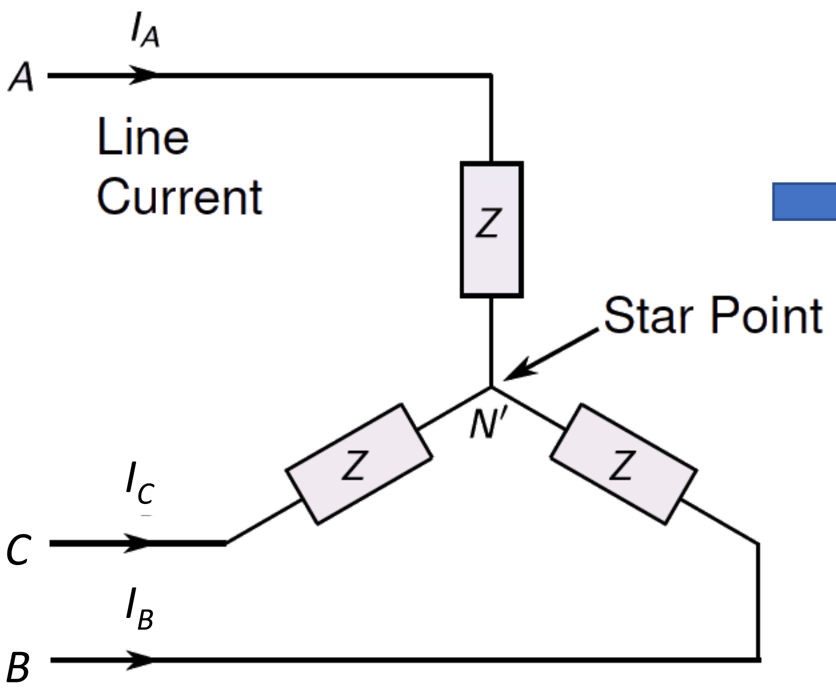
$$Rb = \frac{R3\,Rc}{R2}$$

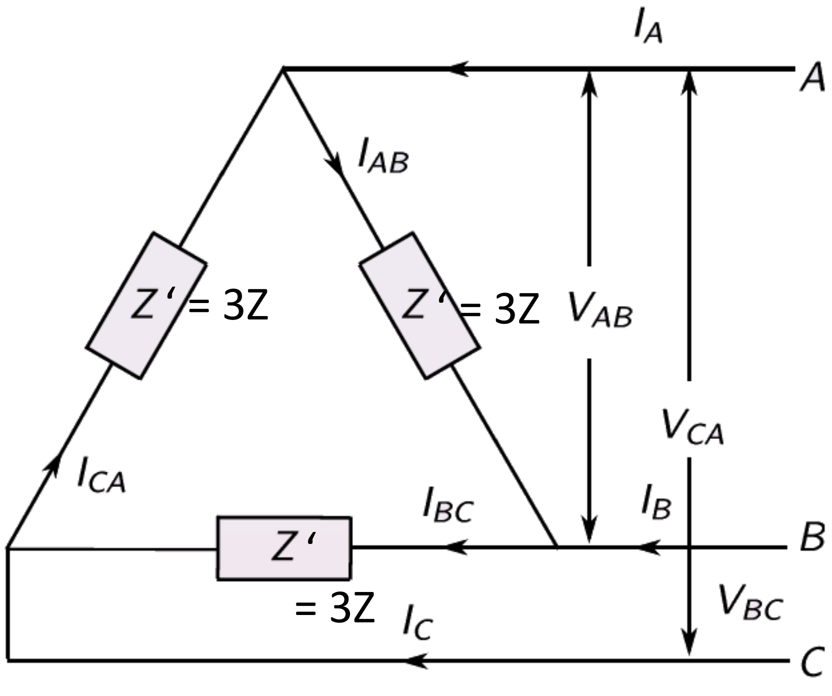
$$R2 = \frac{\frac{R3\,Rc}{R1}Rc}{\frac{R3\,Rc}{R1} + \frac{R3\,Rc}{R2} + Rc}$$

$$R2 = \frac{\frac{R3\,Rc}{R1}}{\frac{R3}{R1} + \frac{R3}{R2} + 1}$$

$$Rc = \frac{R3\,R2 + R3\,R1 + R1\,R2}{R3}$$

Line currents – Transformation for Loads

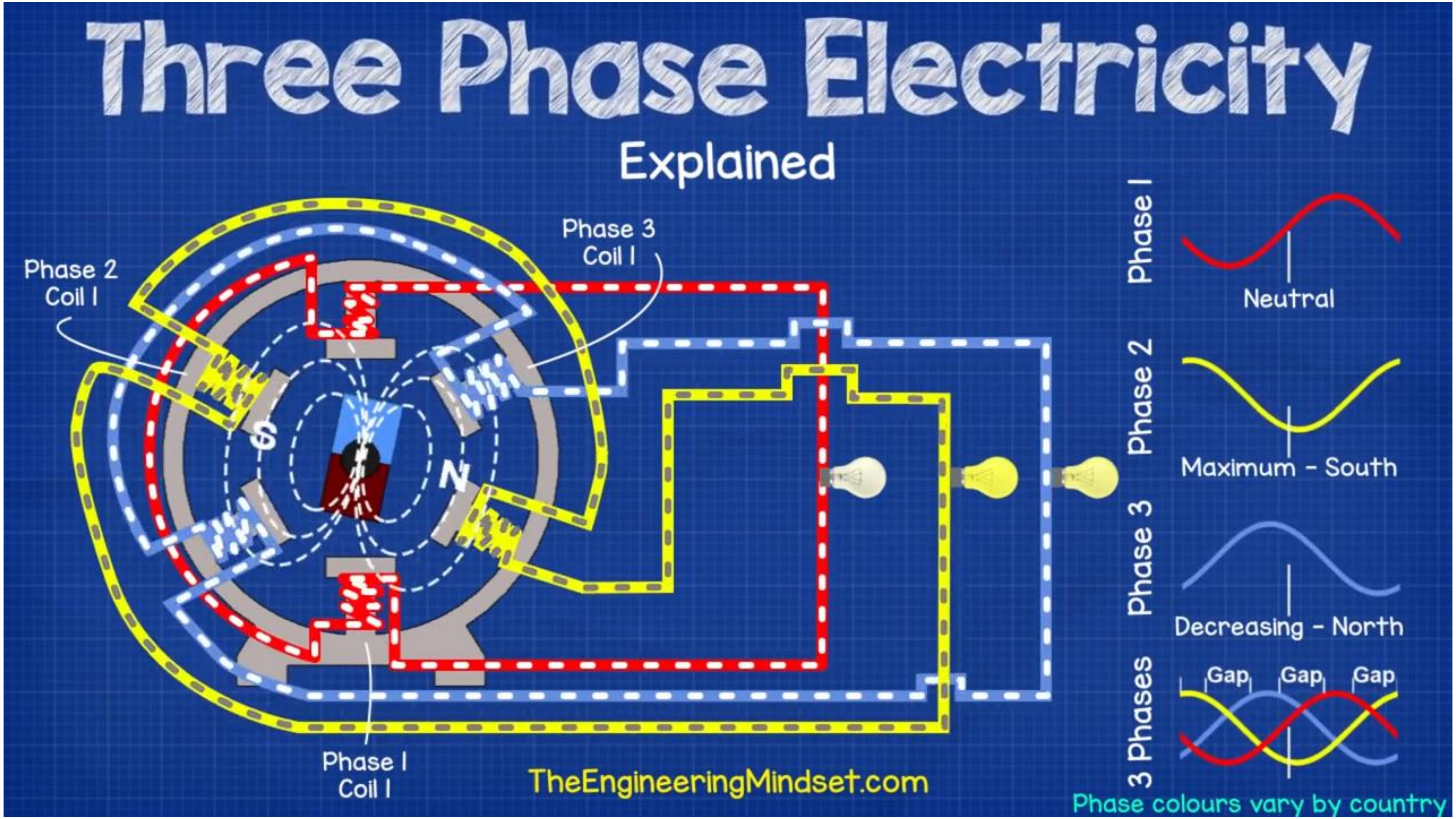




$$I_A = \sqrt{3} I_{AB} \angle\text{-}30°$$

$$\text{or } I_{AB} = \frac{I_A}{\sqrt{3}} \angle 30°$$

Three-phase electricity



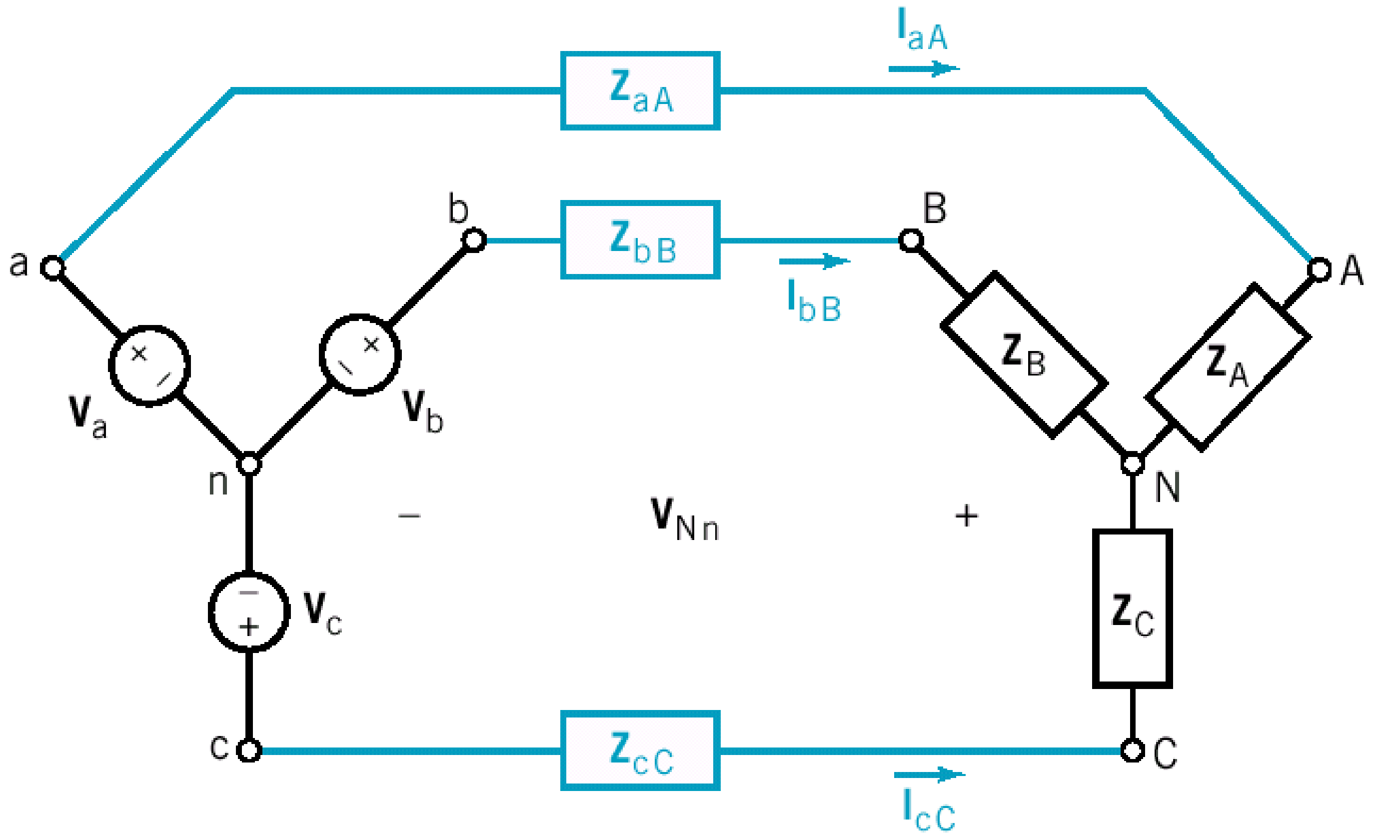
https://www.youtube.com/watch?v=4oRT7PoXSS0&ab_channel=TheEngineeringMindset

Y-to-Y circuit

If the circuit is balanced, V_{Nn} will be 0.

In other words, there will be no returning current from *N* to *n*.

The neutral cable is thus NOT needed!



Suppose $Z_A = Z_B = Z_C = Z\angle\theta$ and $V_{AN} = V_{ph}$ (voltage in rms)

Power in each impedance = $\dfrac{V_{ph}^2}{Z}\cos\theta$

Total power delivered to load = $\dfrac{3V_{ph}^2}{Z}\cos\theta$

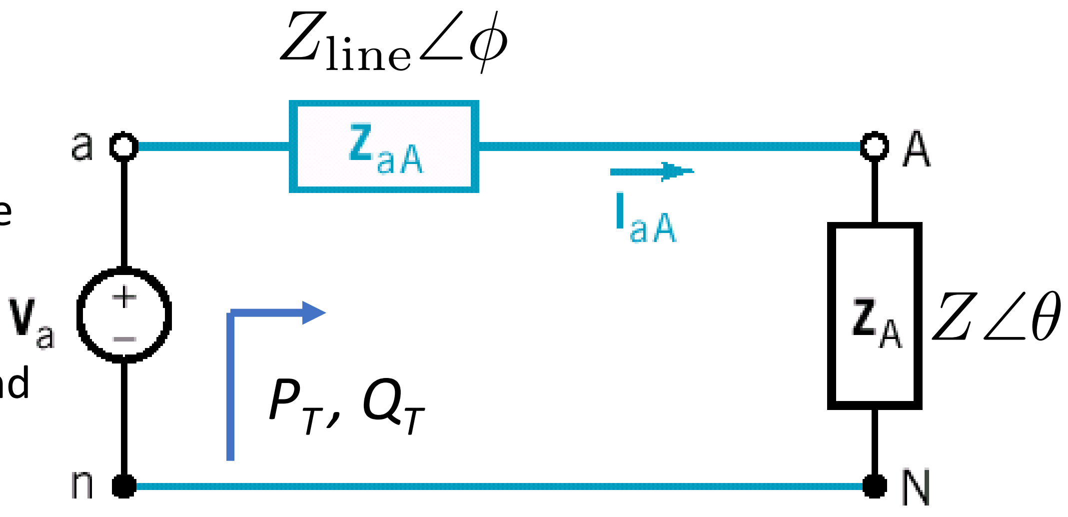
Single-phase equivalent

For a balanced circuit, we know that

- $V_{Nn} = 0$.
- All line currents are equal and differ by 120º.
- Equal power is dissipated in each load.

So, we can simplify the representation to a single-phase circuit, like considering only one of the phases.

Solve the circuit like a single-phase circuit, and then extend all the phase voltages and currents accordingly.



$$Z_T = (Z_{\text{line}}\cos\phi + Z\cos\theta) + j(Z_{\text{line}}\sin\phi + Z\sin\theta)$$

$$|Z_T| = \sqrt{(Z_{\text{line}}\cos\phi + Z\cos\theta)^2 + (Z_{\text{line}}\sin\phi + Z\sin\theta)^2}$$

$$I_{aA} = \frac{V_a}{|Z_T|} \quad \text{(current and voltage in rms)}$$

$$P_T = I_{aA}^2(Z_{\text{line}}\cos\phi + Z\cos\theta)$$

$$Q_T = I_{aA}^2(Z_{\text{line}}\sin\phi + Z\sin\theta)$$

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

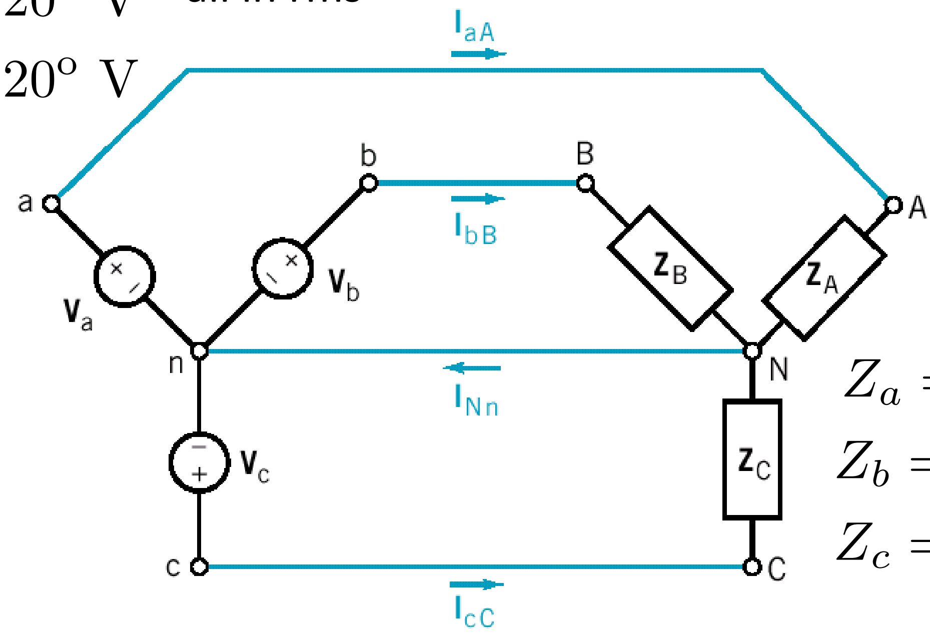
$$\theta_T = \tan^{-1}(Q_T/P_T)$$

Unbalanced loads

What is the apparent power delivered to the load by the source?

$$V_a = 110\angle 0^o \quad \text{V}$$
$$V_b = 110\angle{-120^o} \ \text{V}$$
$$V_c = 110\angle{+120^o} \ \text{V}$$

all in rms



$$Z_a = 50 + j80 \ \Omega$$
$$Z_b = j50 \ \Omega$$
$$Z_c = 100 + j25 \ \Omega$$

$$I_{aA} = \frac{110\angle 0^o}{50 + j80} = 1.16\angle{-58^o} \ \text{A}$$
$$I_{bB} = \frac{110\angle{-120^o}}{j50} = 2.2\angle 150^o \ \text{A}$$
$$I_{cC} = \frac{110\angle 120^o}{100 + j25} = 1.07\angle 106^o \ \text{A}$$

$$I_{Nn} = I_{aA} + I_{bB} + I_{cC} = 1.9556\angle 144.2^o \ \text{A}$$

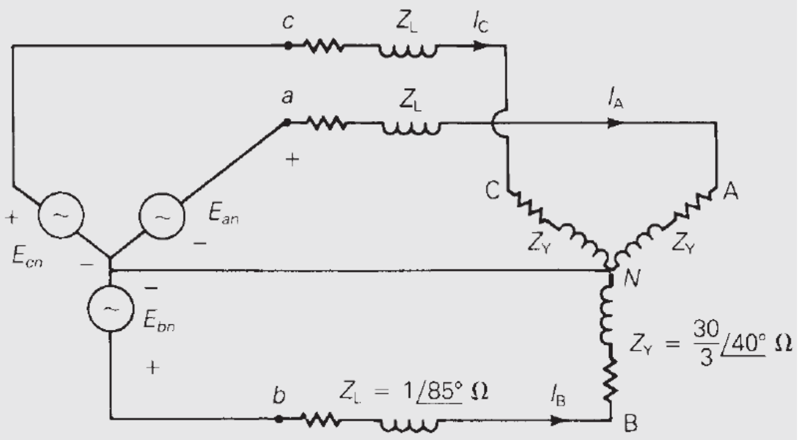
$$S_A = V_a I_{aA}^* = 68 + j109 \ \text{VA}$$
$$S_B = V_b I_{bB}^* = j242 \ \text{VA}$$
$$S_C = V_c I_{cC}^* = 114 + j28 \ \text{VA}$$
$$S = S_A + S_B + S_C = 182 + j379 \ \text{VA}$$

Example: Balanced-Δ and-Y loads

A balanced, positive-sequence, Y-connected voltage source with $E_{ab} = 480\angle 0°$ volts is applied to a balanced-Δ load with $Z_\Delta = 30\angle 40°\ \Omega$. The line impedance between the source and load is $Z_L = 1\angle 85°\ \Omega$ for each phase. Calculate the line currents, the Δ-load currents, and the voltages at the load terminals.

SOLUTION

First, convert the Δ load to an equivalent Y. Then connect the source and Y-load neutrals with a zero-ohm neutral wire. The connection of the neutral wire has no effect on the circuit, since the neutral current I_n=0 in a balanced system.



$$I_A = \frac{E_{an}}{Z_L + Z_Y} = \frac{\frac{480}{\sqrt{3}}\angle -30°}{1\angle 85° + \frac{30}{3}\angle 40°}$$

$$= \frac{277.1\angle -30°}{(0.0872 + j0.9962) + (7.660 + j6.428)}$$

$$= \frac{277.1\angle -30°}{(7.748 + j7.424)} = \frac{277.1\angle -30°}{10.73\angle 43.78°} = 25.83\angle -73.78°\ \text{A}$$

$$I_B = 25.83\angle 166.22°\ \text{A}$$

$$I_C = 25.83\angle 46.22°\ \text{A}$$

$$I_{AB} = \frac{I_a}{\sqrt{3}}\angle +30° = \frac{25.83}{\sqrt{3}}\angle -73.78° + 30° = 14.91\angle -43.78°\ \text{A}$$

$$I_{BC} = 14.91\angle -163.78°\ \text{A}$$

$$I_{CA} = 14.91\angle +76.22°\ \text{A}$$

The voltages at the load terminals are

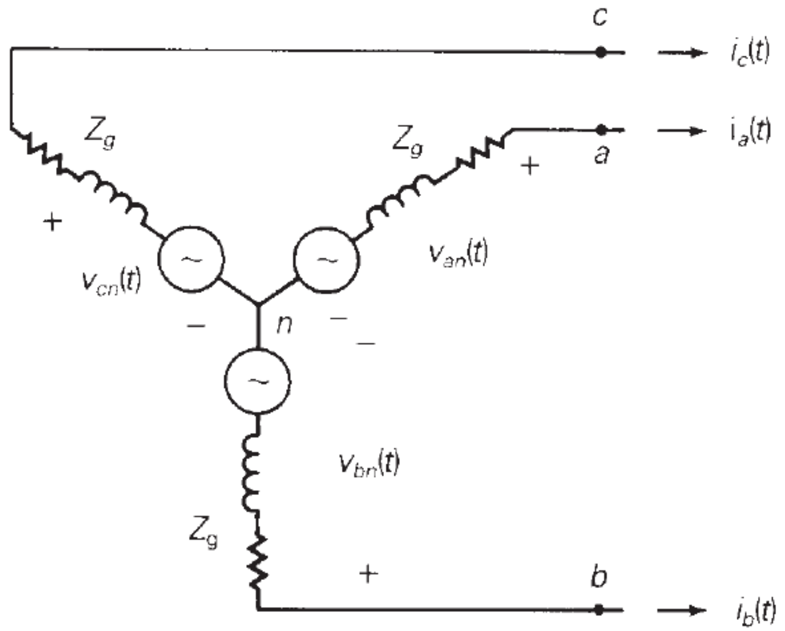
$$E_{AB} = Z_\Delta I_{AB} = (30\angle 40°)(14.91\angle -43.78°) = 447.3\angle -3.78°$$

$$E_{BC} = 447.3\angle -123.78°$$

$$E_{CA} = 447.3\angle 116.22°\ \text{volts}$$

Instantaneous power (updated slide)

Balanced three-phase circuits



$$v_{an}(t) = \sqrt{2}V_{LN}\cos(\omega t + \delta) \quad \text{volts}$$

$$i_a(t) = \sqrt{2}I_L \cos(\omega t + \beta) \ \ A$$

$$\begin{aligned} p_a(t) &= v_{an}(t)i_a(t) \\ &= 2V_{LN}I_L \cos(\omega t + \delta)\cos(\omega t + \beta) \\ &= V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L \cos(2\omega t + \delta + \beta) \quad W \end{aligned}$$

$$\begin{aligned} p_b(t) &= 2V_{LN}I_L\cos(\omega t + \delta - 120°)\cos(\omega t + \beta - 120°) \\ &= V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L \cos(2\omega t + \delta + \beta - 240°) \end{aligned}$$

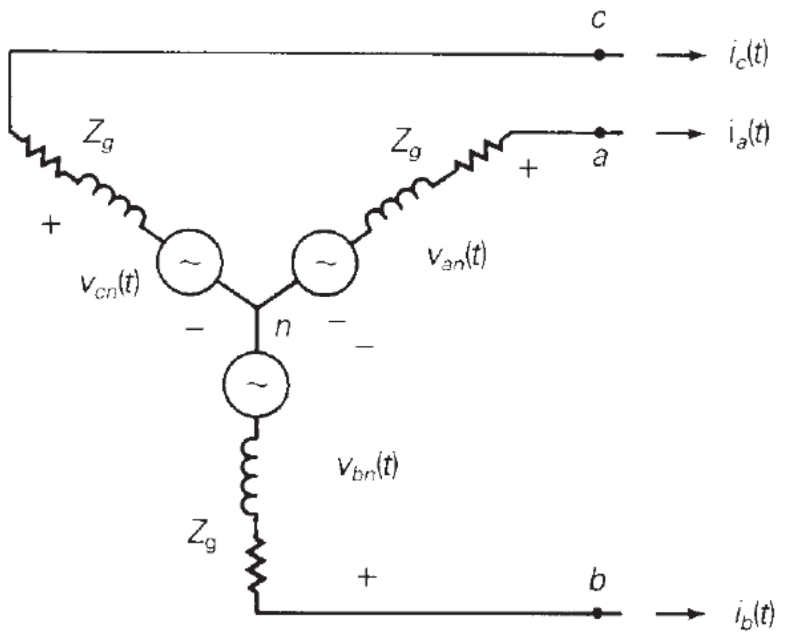
$$\begin{aligned} p_c(t) &= 2V_{LN}I_L\cos(\omega t + \delta + 120°)\cos(\omega t + \beta + 120°) \\ &= V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L \cos(2\omega t + \delta + \beta + 240°) \quad W \end{aligned}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\boxed{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$\boxed{\begin{aligned} p_{3\phi}(t) &= p_a(t) + p_b(t) + p_c(t) \\ &= 3V_{LN}I_L \cos(\delta - \beta) + V_{LN}I_L[\cos(2\omega t + \delta + \beta) \\ &\quad + \cos(2\omega t + \delta + \beta - 240°) \\ &\quad + \cos(2\omega t + \delta + \beta + 240°)] \ \ W \end{aligned}}$$

(Average) Power (updated slide)



$$P_{3\phi} = 3V_{LN}I_L\cos(\delta - \beta) \quad W$$

$$V_{LN} = V_{LL}/\sqrt{3} \quad \text{and} \quad P_{3\phi} = \sqrt{3}V_{LL}I_L\cos(\delta - \beta) \quad W$$

$$\boxed{\cos\phi}$$

$$P_{3\phi} = 3V_{LN}I_L\boxed{\cos\phi}$$

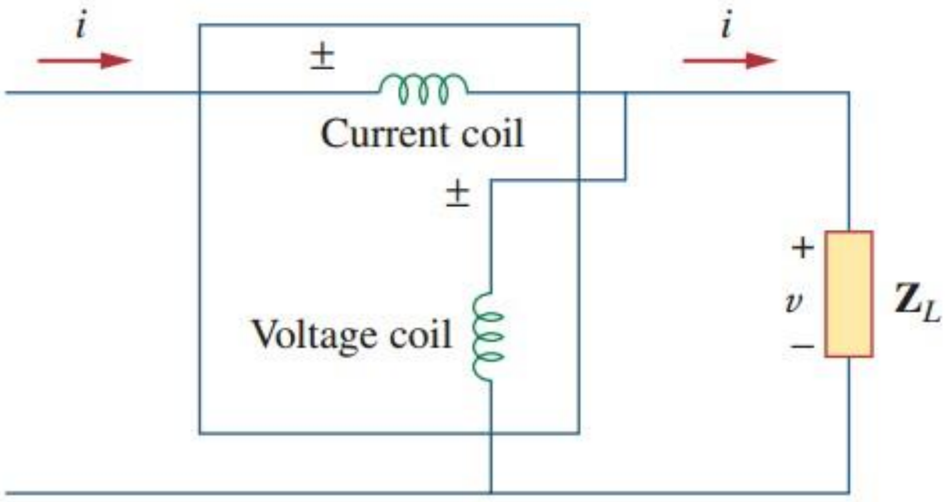
$$P_{3\phi} = \sqrt{3}V_{LL}I_L\boxed{\cos\phi}$$

$$\cos\theta\,\cos\varphi = \frac{\cos(\theta-\varphi)+\cos(\theta+\varphi)}{2}$$

$$\sin\theta\,\sin\varphi = \frac{\cos(\theta-\varphi)-\cos(\theta+\varphi)}{2}$$

V and I in rms values

Wattmeter



Current coil picks up the current through
Voltage coil picks up the voltage across
Meter reading gives the average product.

$$P = P_{\text{average}} = VI\cos\phi$$

where ϕ is the angle between V and I

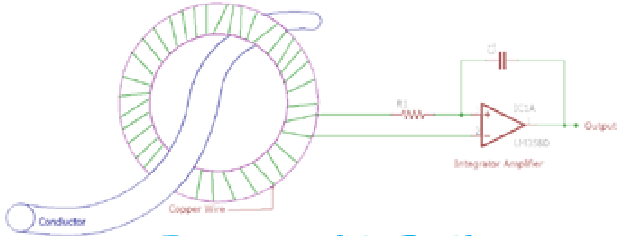
Wattmeter: Current sensing techniques

Measurement methods:

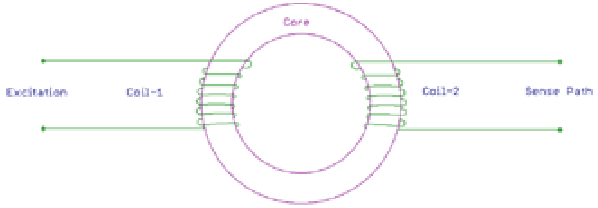
1. Resistive (direct)
   a. Shunt resistors

2. Magnetic (indirect)
   a. Current transformer
   b. Rogowski coil
   c. Hall effect device- hall sensor

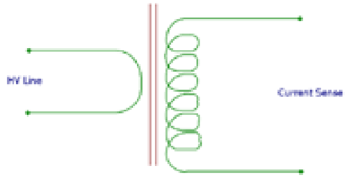
3. Transistor (direct)
   a. RDS(ON)
   b. Ratio-metric



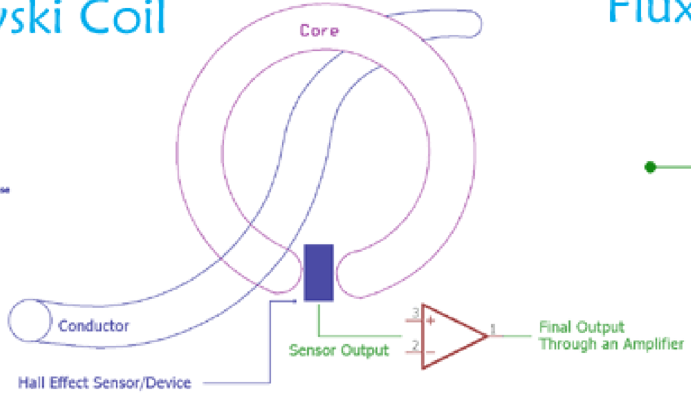
Rogowski Coil



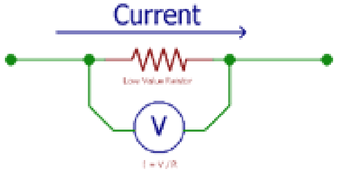
Flux Gate



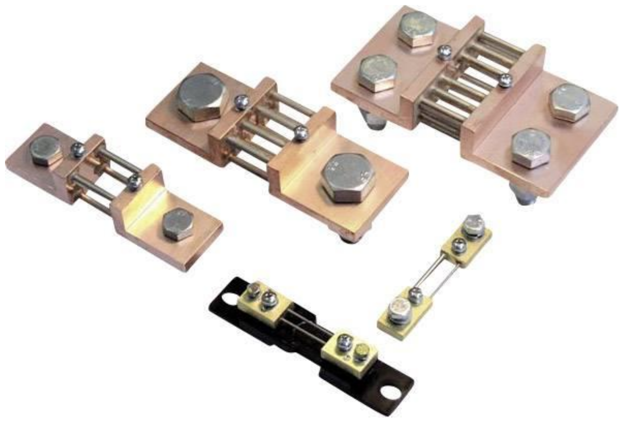
Current Transformer

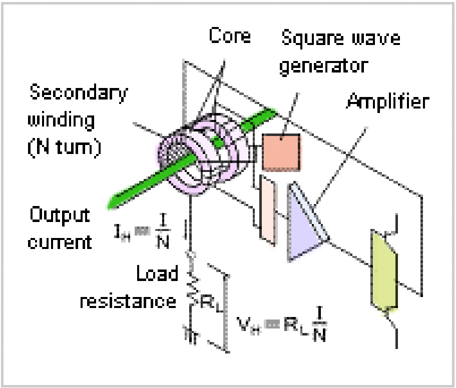


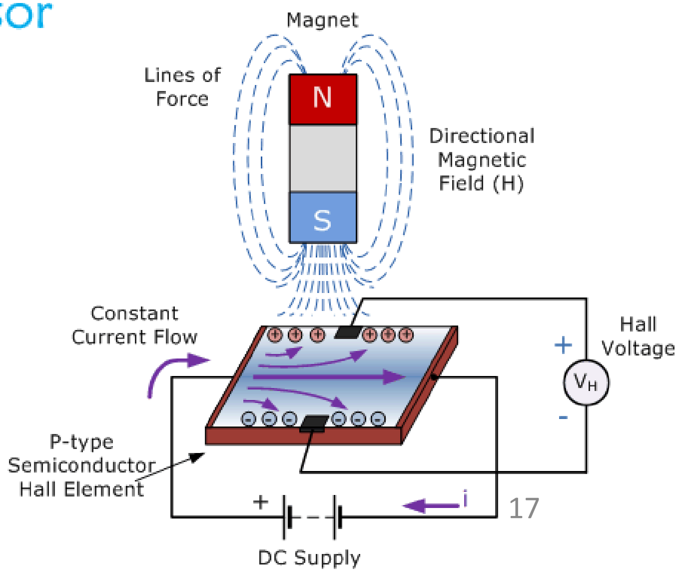
Hall Sensor



Shunt Resistor

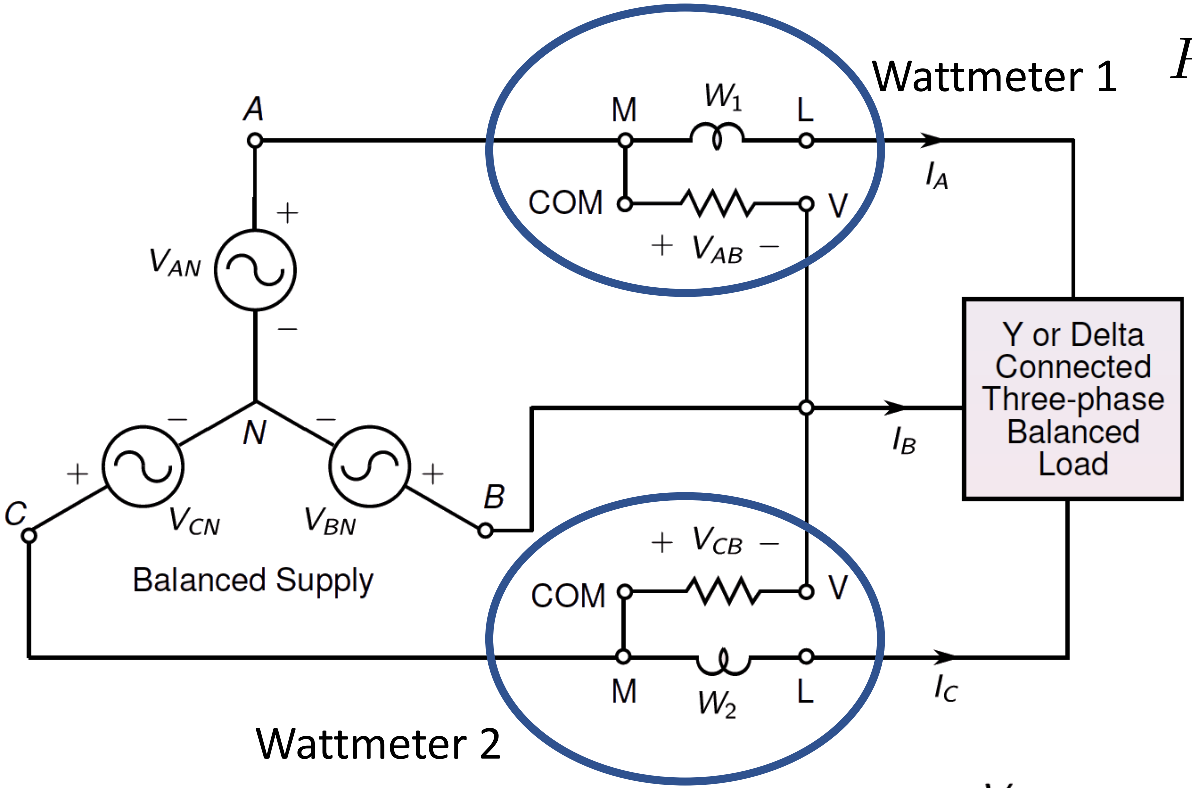




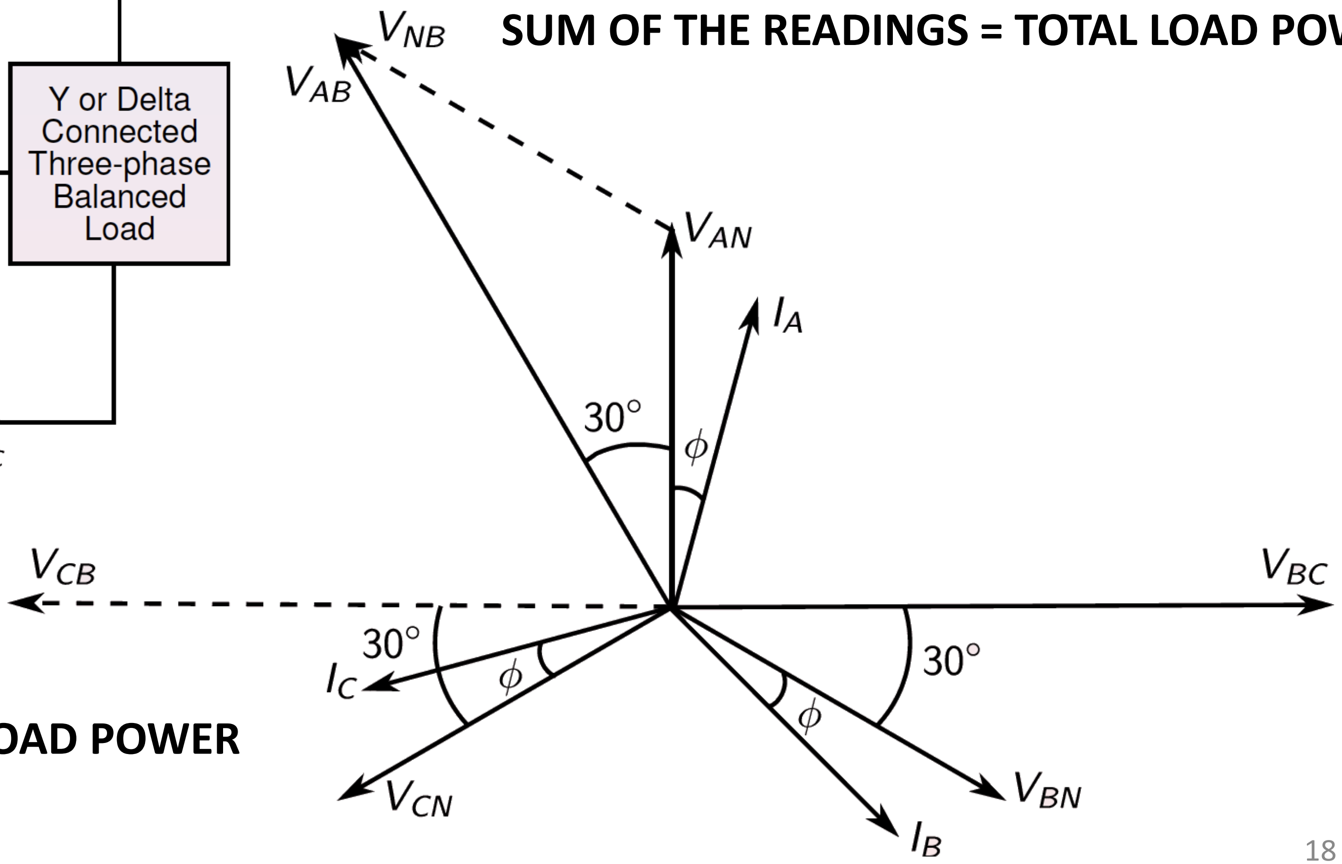


Measuring power

$$P_1 = |V_{AB}||I_A|\cos(\phi + 30^o) = |V||I|\cos(\phi + 30^o)$$
$$P_2 = |V_{BC}||I_C|\cos(30^o - \phi) = |V||I|\cos(30^o - \phi)$$
$$P_1 + P_2 = |V||I|(\cos(\phi + 30^o) + \cos(30^o - \phi))$$
$$= 2|V||I|\cos 30^o \cos\phi = \sqrt{3}|V||I|\cos\phi$$

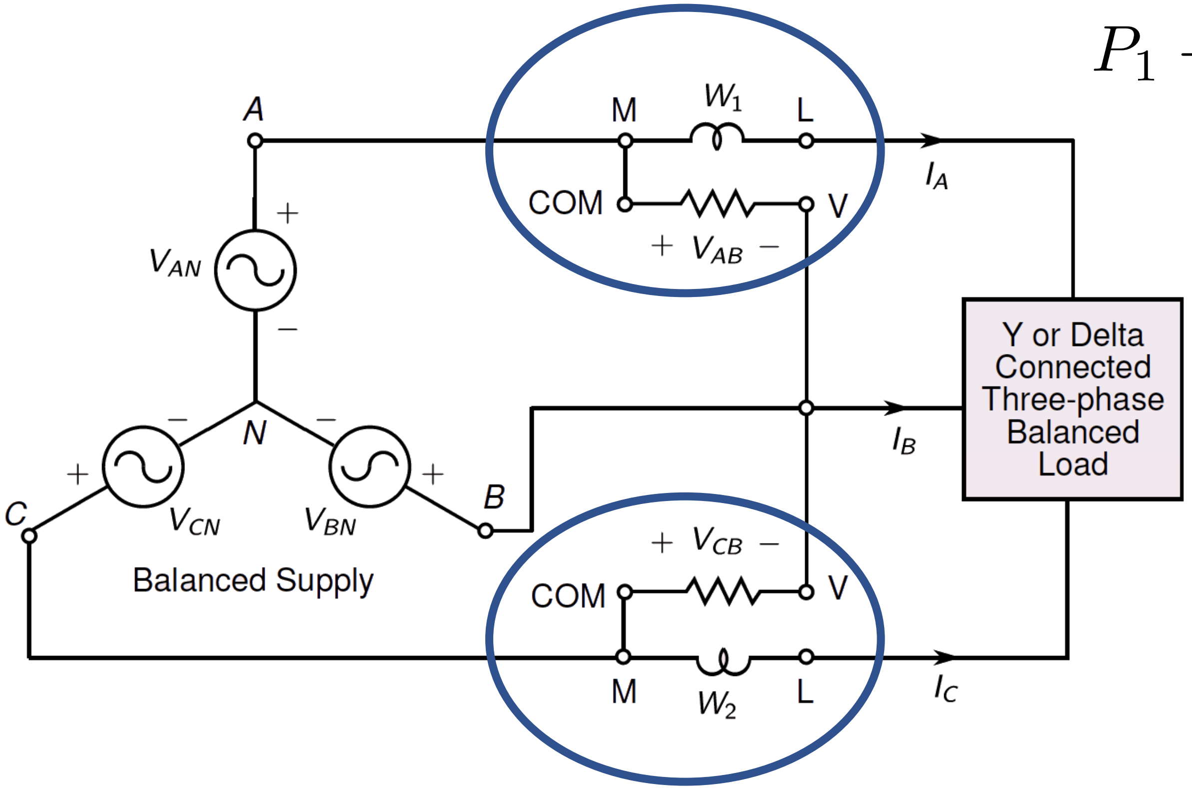


SUM OF THE READINGS = TOTAL LOAD POWER



SUM OF THE READINGS W1 and W2 = TOTAL LOAD POWER

Impedance angle



$$P_1 - P_2 = |V||I|(\cos(\phi + 30^o) - \cos(30^o - \phi))$$
$$= -2|V||I|\sin 30^o \sin\phi = -|V||I|\sin\phi$$

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{-\tan\phi}{\sqrt{3}}$$

$$\tan\phi = -\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2} = \sqrt{3}\frac{P_2 - P_1}{P_1 + P_2}$$

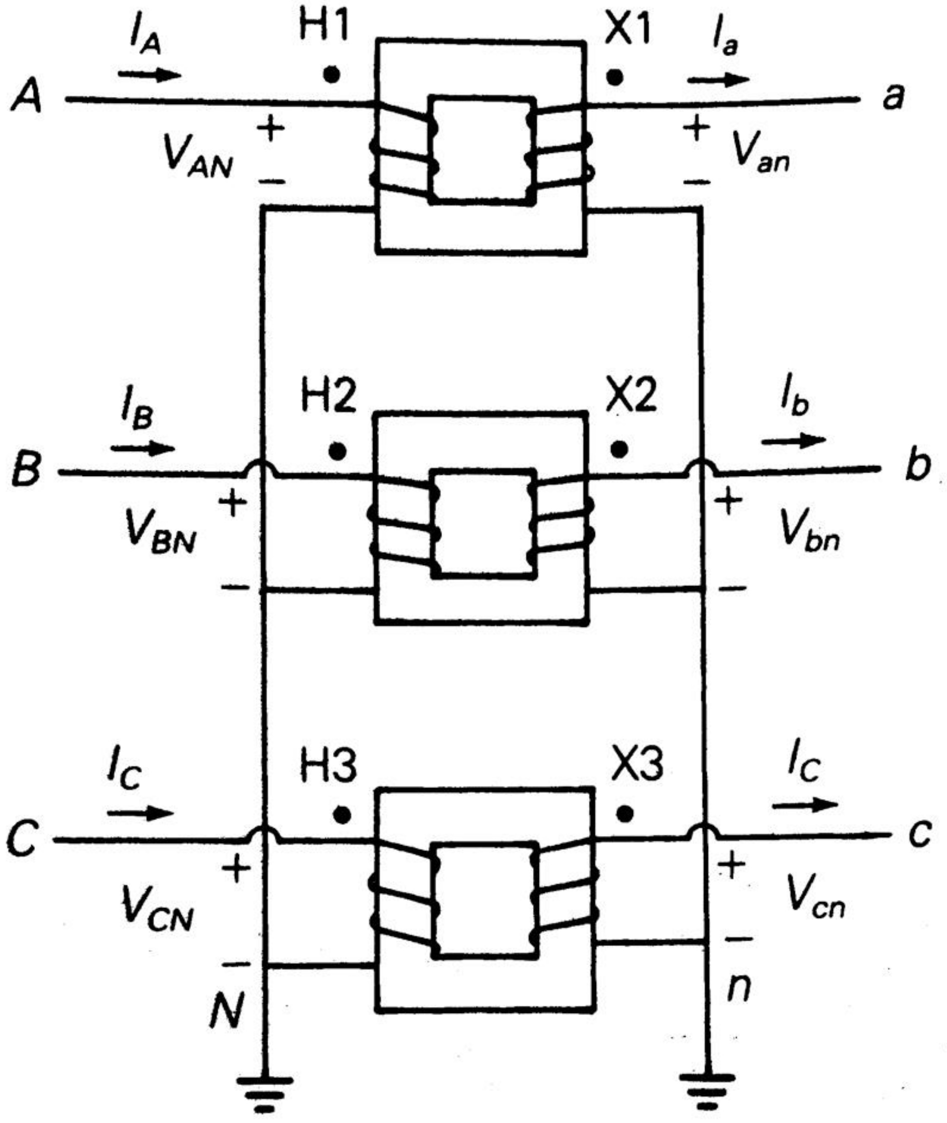
IMPEDANCE ANGLE CAN ALSO BE FOUND

$$\boxed{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

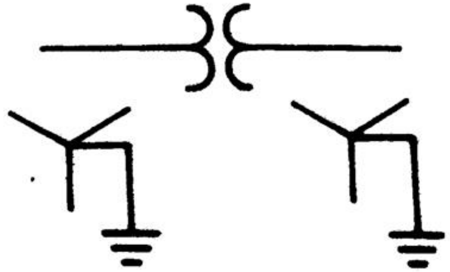
Three-phase transformer arrangement

Y-Y connection

Primary and secondary voltages are in phase



Y-connection provides a neutral line which is good for transmission as it allows unbalanced current to flow out and simplify insulation.



Three-phase transformer arrangement

Y-Δ connection

- Y side to N is leading Δ side to n by 30ᵒ in this case.
- Δ-connection suppresses third harmonic currents that are generated due to the nonlinear core characteristics (hysteresis) by trapping the harmonic currents in the Δ loop.
- Single transformer: $\frac{V_{AN}}{V_{ab}} = \frac{N_{H1}}{N_{X1}}$

In single phase equivalent circuit, the Δ side quantities

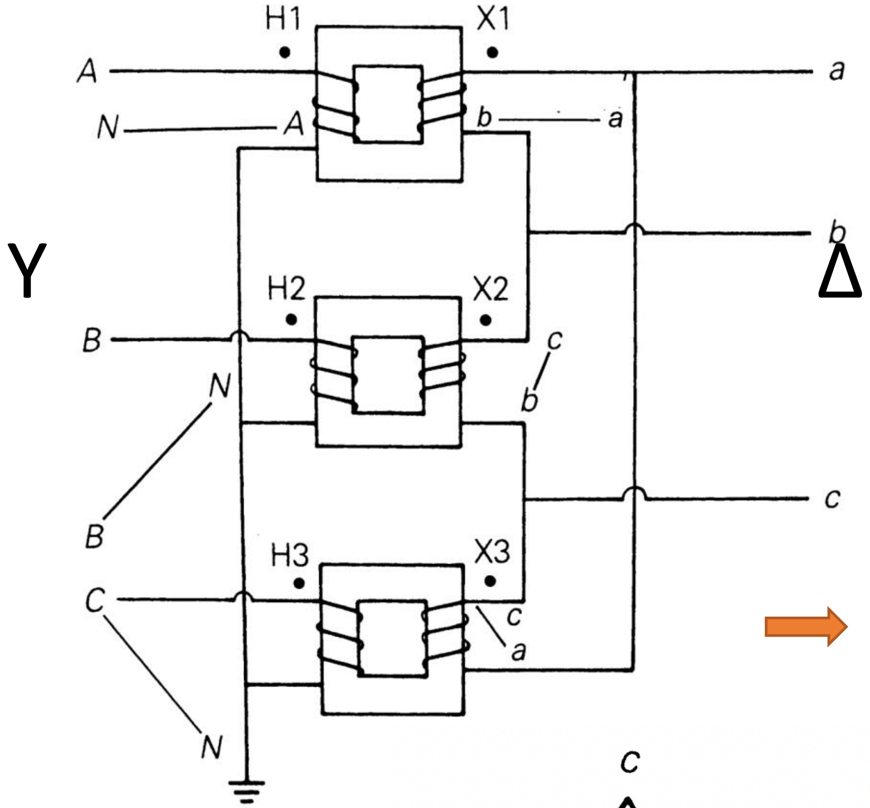
$V_{an} = \frac{V_{ab}}{\sqrt{3}}\underline{/\text{-}30^{o}}$

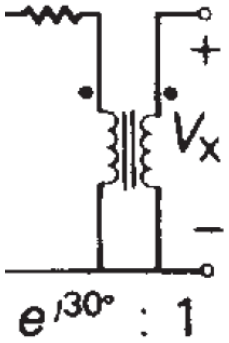
or

$I_a = \sqrt{3} I_{ab}\underline{/\text{-}30^{o}}$

Thus, we have: $V_{AN} = \sqrt{3}\frac{N_{H1}}{N_{X1}} V_{an}\underline{/30^{o}}$

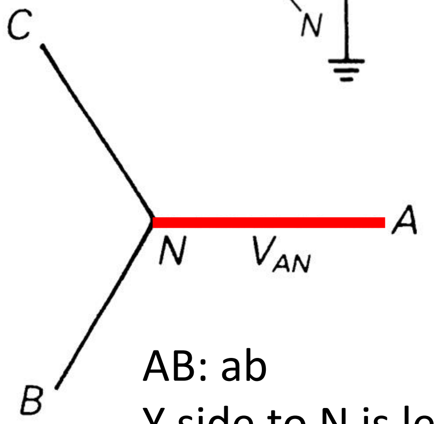
For p.u. calculation, $\sqrt{3}\frac{N_{H1}}{N_{X1}}$ will be eliminated.

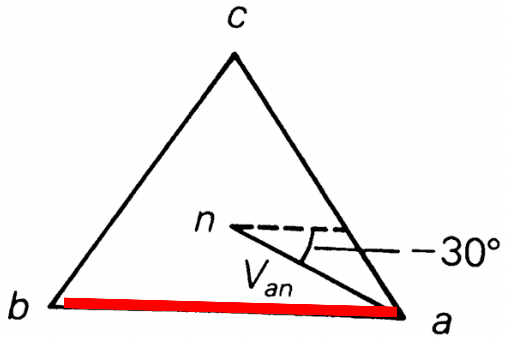




Equivalent model

Y-Δ connection (p.u.)





AB: ab

Y side to N is leading Δ side to n by 30ᵒ in this case.

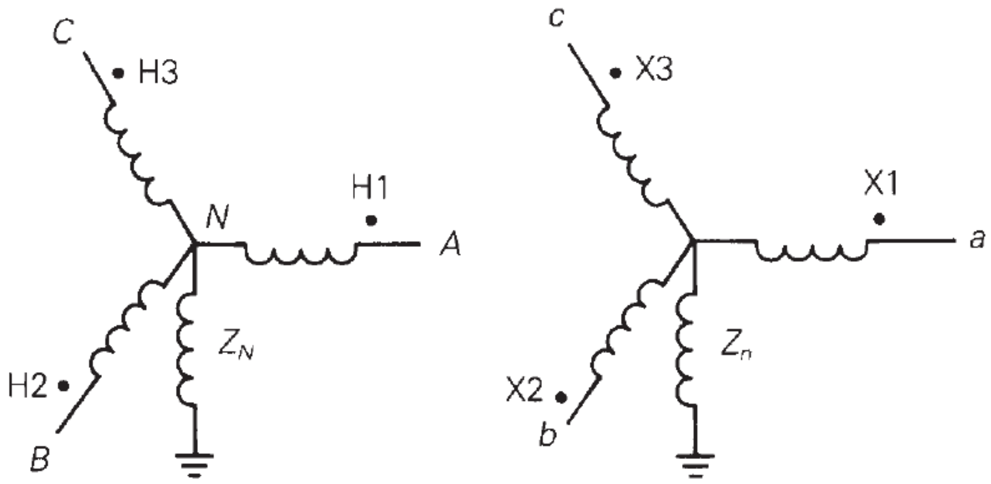
Per unit for three-phase transformer

Y-Y connection

The following are two conventional rules for selecting base quantities:

- A common S_{base} is selected for both the H and X terminals.
- The ratio of the voltage bases V_{baseH}/V_{baseX} is selected to be equal to the ratio of the rated line-to-line voltages $V_{ratedHLL}/V_{ratedXLL}$.

For the American standard, the positive-sequence (A-B-C) voltages and currents on the high-voltage side of the Y–Δ transformer lead the corresponding quantities on the low-voltage side by 30°.



Example: Voltage calculations

- Three single-phase two-winding transformers, each rated 400 MVA, 13.8/199.2 kV, with leakage reactance X_{eq}=0.10 per unit, are connected to form a three-phase bank. Winding resistances and exciting current are neglected. The high-voltage windings are connected in Y. A three-phase load operating under balanced positive- sequence conditions on the high-voltage side absorbs 1000 MVA at 0.90 p.f. lagging, with $V_{AN}=199.2\angle 0^\circ$ kV. Determine the voltage V_{an} at the low-voltage bus if the low-voltage windings are connected (a) in Y and (b) in Δ.

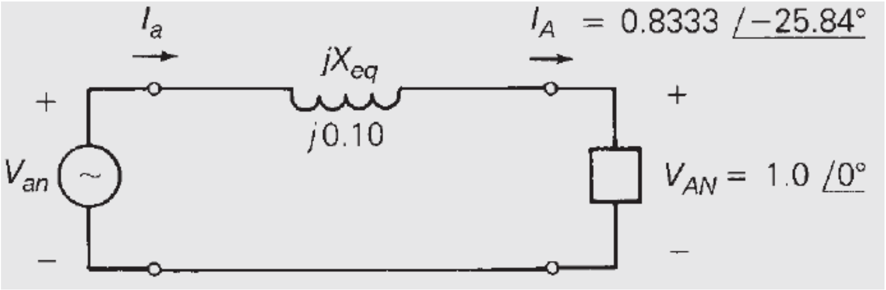
SOLUTION for (a)

Using the transformer bank ratings as base quantities,
$S_{base3\Phi}$=1200 MVA, $V_{baseHLL}$=345 kV, and I_{baseH}= 1200/(345$\sqrt{3}$) =2.008 kA.
The per-unit load voltage and load current are then

$$V_{AN} = 1.0\angle 0^\circ \quad \text{per unit}$$

$$I_A = \frac{1000/(345\sqrt{3})}{2.008}\angle -\cos^{-1}0.9 = 0.8333\angle -25.84^\circ \quad \text{per unit}$$

$$I_a = I_A = 0.8333\angle -25.84^\circ \quad \text{per unit}$$



Y connected winding

$$\begin{aligned} V_{an} &= V_{AN} + (jX_{eq})I_A \\ &= 1.0\angle 0^\circ + (j0.10)(0.8333\angle -25.84^\circ) \\ &= 1.0 + 0.08333\angle 64.16^\circ = 1.0363 + j0.0750 = 1.039\angle 4.139^\circ \\ &= 1.039\angle 4.139^\circ \text{ per unit} \end{aligned}$$

Further, since $V_{baseXLN}$ =13.8 kV for the low-voltage Y windings, V_{an} = 1.039(13.8) = 14.34 kV, and

$$V_{an} = 14.34\angle 4.139^\circ \quad \text{kV}$$

Example: Voltage calculations

❖ Three single-phase two-winding transformers, each rated 400 MVA, 13.8/199.2 kV, with leakage reactance X_{eq}=0.10 per unit, are connected to form a three-phase bank. Winding resistances and exciting current are neglected. The high-voltage windings are connected in Y. A three-phase load operating under balanced positive- sequence conditions on the high-voltage side absorbs 1000 MVA at 0.90 p.f. lagging, with V_{AN}=199.2$\underline{/0°}$ kV. Determine the voltage V_{an} at the low-voltage bus if the low-voltage windings are connected (a) in Y and (b) in Δ.

SOLUTION for (b)

* Y side to N is leading Δ side to n by 30° in this case.

$$E_{an} = 1.0\underline{/-30°} \quad \text{per unit}$$

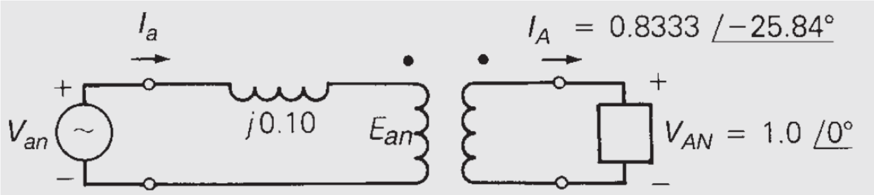
$$I_a = 0.8333\underline{/-25.84° - 30°} = 0.8333\underline{/-55.84°} \quad \text{per unit}$$

$$V_{an} = E_{an} + (jX_{eq})I_a = 1.0\underline{/-30°} + (j0.10)(0.8333\underline{/-55.84°})$$

$$V_{an} = 1.039\underline{/-25.861°} \quad \text{per unit}$$

Further, since $V_{baseXLN} = 13.8/\sqrt{3} = 7.967$ kV for the low-voltage Δ windings, $V_{an} = (1.039)(7.967) = 8.278$ kV, and

$$V_{an} = 8.278\underline{/-25.861°} \quad \text{kV}$$



Δ connected winding

Summary

- Three-phase voltage source and load
- Y and Δ connections
- Phase and line values
- Power in three-phase loads
- Wattmeter and measurements
- 3ϕ transformer connections