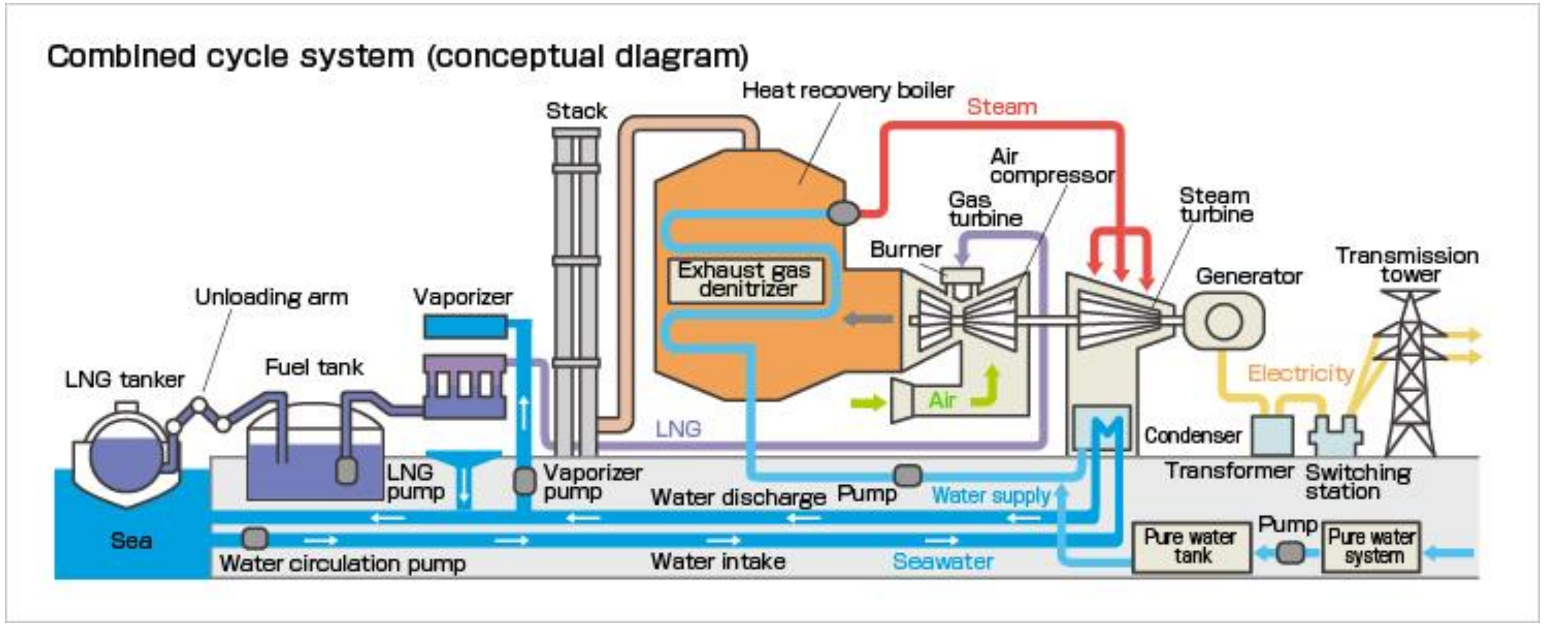
EE3123 Introduction to Electric Power Systems

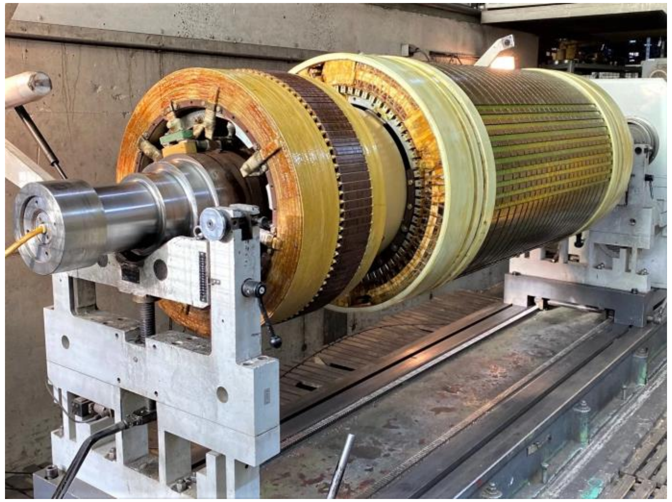
Power System Stability

Prof. CQ Jiang

Many thanks to Prof. Michael Tse

Synchronous generators






Nuclear plant

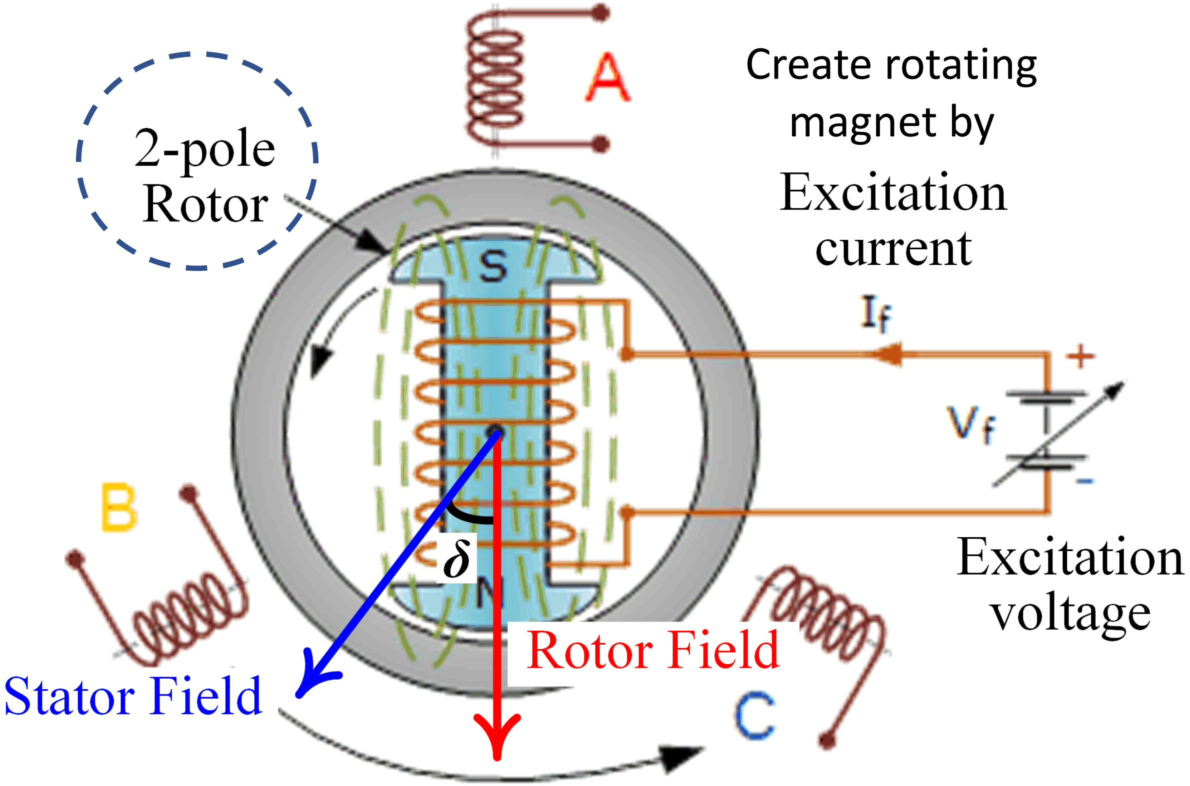

Hydro plant


Coal-fired plant

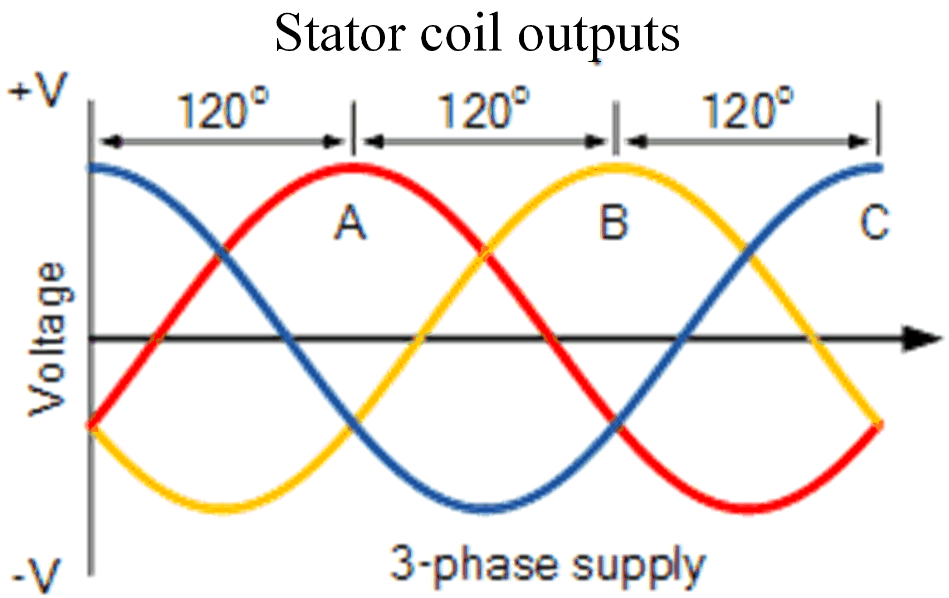
Synchronous generators

https://www.youtube.com/watch?v=SPg7hOxFItI

Synchronous generators

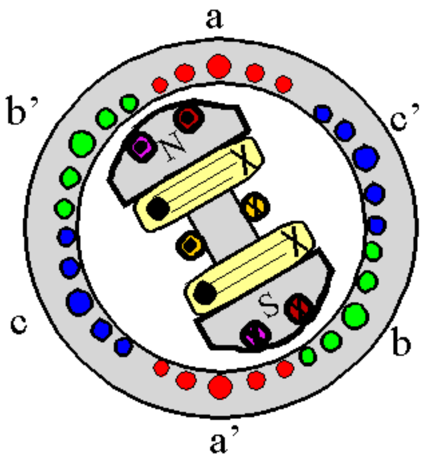


Three Stator windings 120° apart

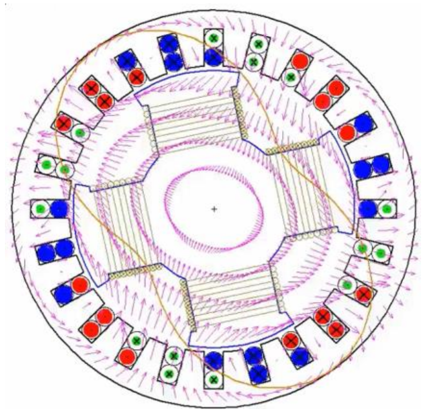


Mechanical power $\xrightarrow{\text{Synchronous generator}}$ Electrical power

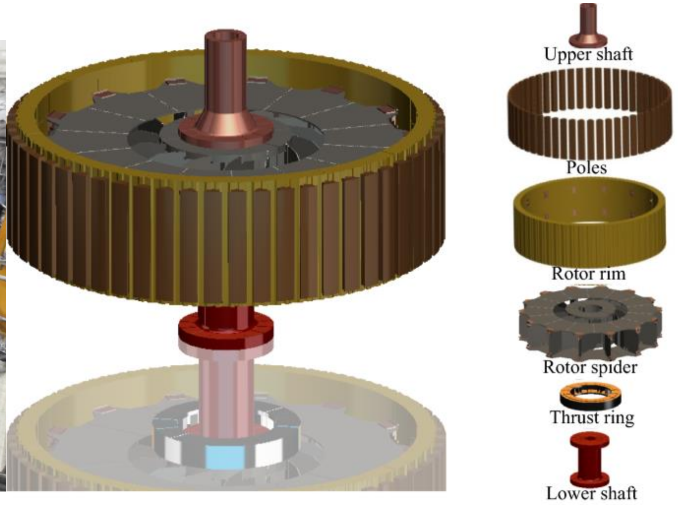
Salient-pole machine



2-pole



4-pole (p-pole)



rotor synchronous speed
= electrical synchronous speed
= 2 π (frequency)

rotor synchronous speed
reduced by half or (2/p)
= 2 π (frequency) (2/p)

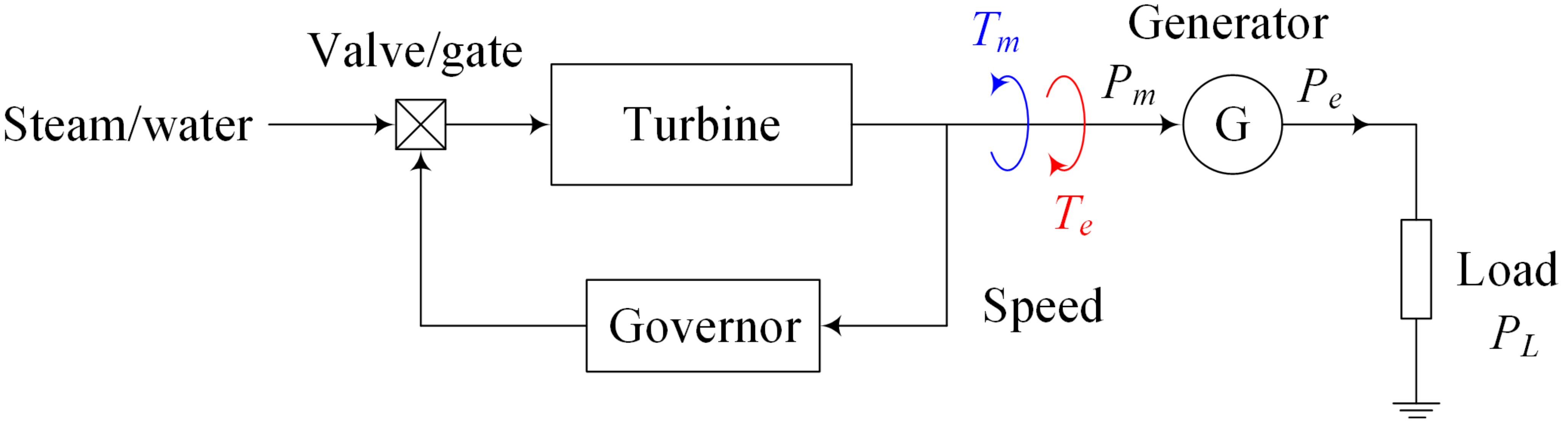
Electrical synchronous speed = 2 π 60 = 377 rad/s
For 16-pole system,
Rotor synchronous speed = 377 (2/16) = 47.12 rad/s

Linear Motion			Rotation		
Quantity	Symbol/ Equation	MKS unit	Quantity	Symbol/ Equation	MKS unit
Length	s	meter (m)	Angular displacement	θ	radian (rad)
Mass	M	kilogram (kg)	Moment of inertia	$J=\int r^2 dm$	$kg{\cdot}m^2$
Velocity	$v=ds/dt$	meter/second (m/s)	Angular velocity	$\omega=d\theta/dt$	rad/s
Acceleration	$a=dv/dt$	m/s^2	Angular acceleration	$\alpha=d\omega/dt$	rad/s^2
Force	$F=Ma$	newton (N)	Torque	$T=J\alpha$	newton-meter (N·m) or J/rad
Work	$W=\int F ds$	joule (J)	Work	$W=\int T d\theta$	J, or W·s
Power	$p=dW/dt$ $=Fv$	watt (W)	Power	$p=dW/dt$ $=T\omega$	W

defined as the quantity expressed by the body resisting angular acceleration

Second Newton's law of motion

Mechanical torque: swing equation



T_m = mechanical torque $\quad$ T_e = electrical torque

P_m = mechanical power $\quad$ P_e = electrical power

Newton's second law of motion

$$J\alpha = T_m - T_e$$

Moment of inertia

Angular acceleration

Newton's law
F = ma

Some math! Swing equation

Since $J\alpha = T_m - T_e$ and $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\delta}{\mathrm{d}t^2}$ ← angular position of the rotor field relative to stator field

Type of generating unit		H
Thermal unit	3600 r/min (2-pole)	2.5 ~ 6
	1800 r/min (4-pole)	4 ~ 10
Hydraulic unit		2 ~ 4

we get

$$J\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = T_m - T_e$$

$$J\omega_0\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = T_m\omega_0 - T_e\omega_0 \approx P_m - P_e$$

Torque x speed = power

$$\frac{2H}{\omega_0}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_{m,\mathrm{pu}} - P_{e,\mathrm{pu}} \underbrace{- D(\omega - \omega_0)}_{\text{damping}}$$

Therefore

$$\frac{2S_N}{\omega_0}\left(\frac{J\omega_0^2}{2S_N}\right)\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_m - P_e$$

per unit inertia constant

$$H = \frac{J\omega_0^2}{2S_N}$$

$$\frac{2HS_N}{\omega_0}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_m - P_e$$

$$P_{m,\mathrm{pu}} = \frac{P_m}{S_N}$$

$$P_{e,\mathrm{pu}} = \frac{P_e}{S_N}$$

$$\frac{2H}{\omega_0}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_{m,\mathrm{pu}} - P_{e,\mathrm{pu}}$$

$$\mathrm{H} = \frac{\text{stored kinetic energy at synchronous speed}}{\text{generator voltampere rating}}$$

Mechanical torque: swing equation

$$\boxed{\frac{2H}{\omega_0}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_{m,\mathrm{pu}} - P_{e,\mathrm{pu}} - D(\omega - \omega_0)}$$

Neglecting damping *D* and integrating twice:

$$\frac{d\delta}{dt} = \frac{\omega_0}{2H}(P_m - P_e)t + 0$$

assuming the speed is stable at *t* = 0.

$$\delta(t) = \frac{\omega_0}{4H}(P_m - P_e)t^2 + \delta_0$$

$\delta = \delta_0$ at t = 0.

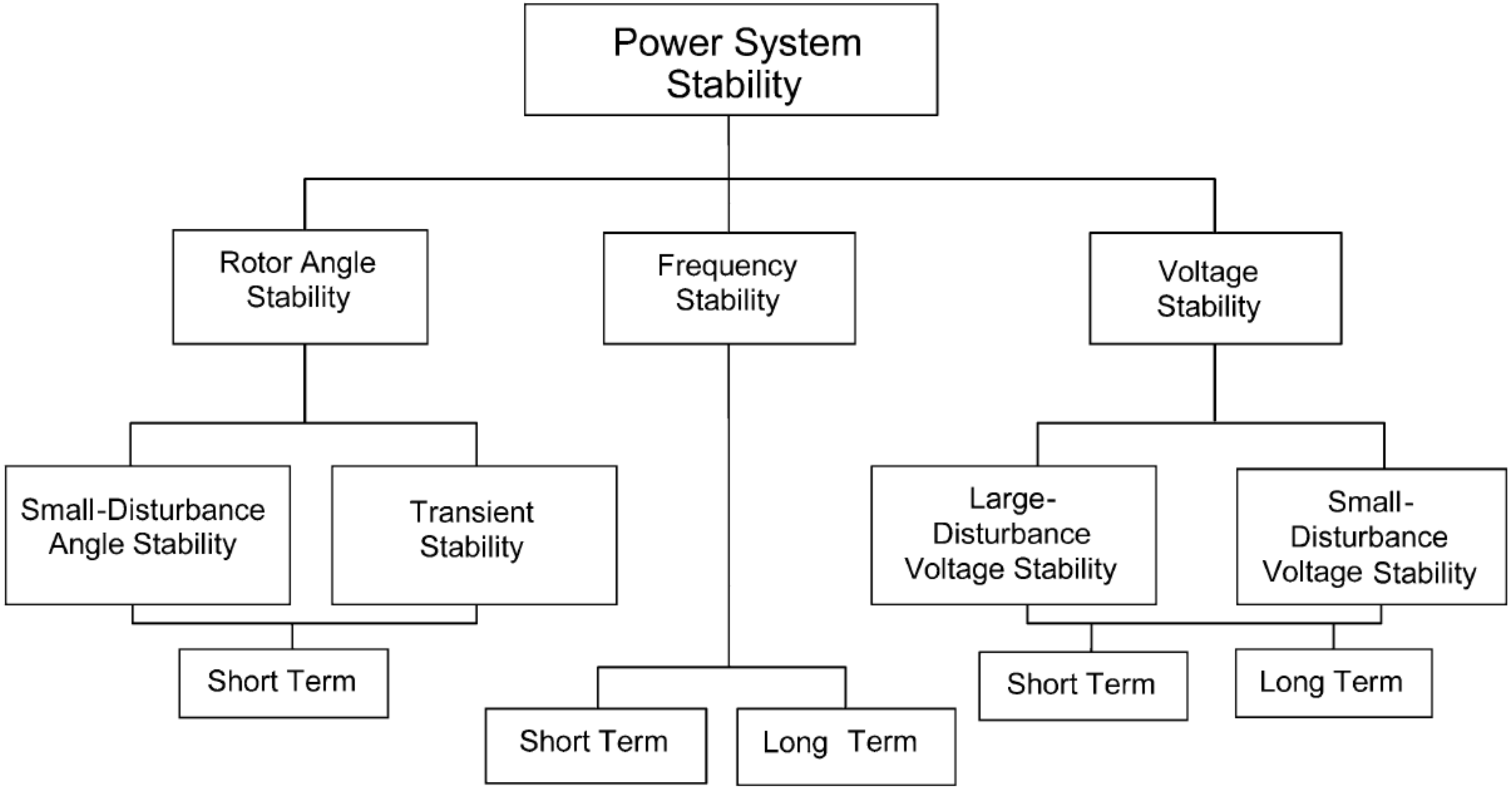
Special case: if generator output = 0, e.g., after a fault occurs:

$$\frac{d\delta(t)}{dt} = \frac{\omega_0 P_m}{2H}t + 0$$

$$\delta(t) = \frac{\omega_0 P_m}{4H}t^2 + \delta_0$$

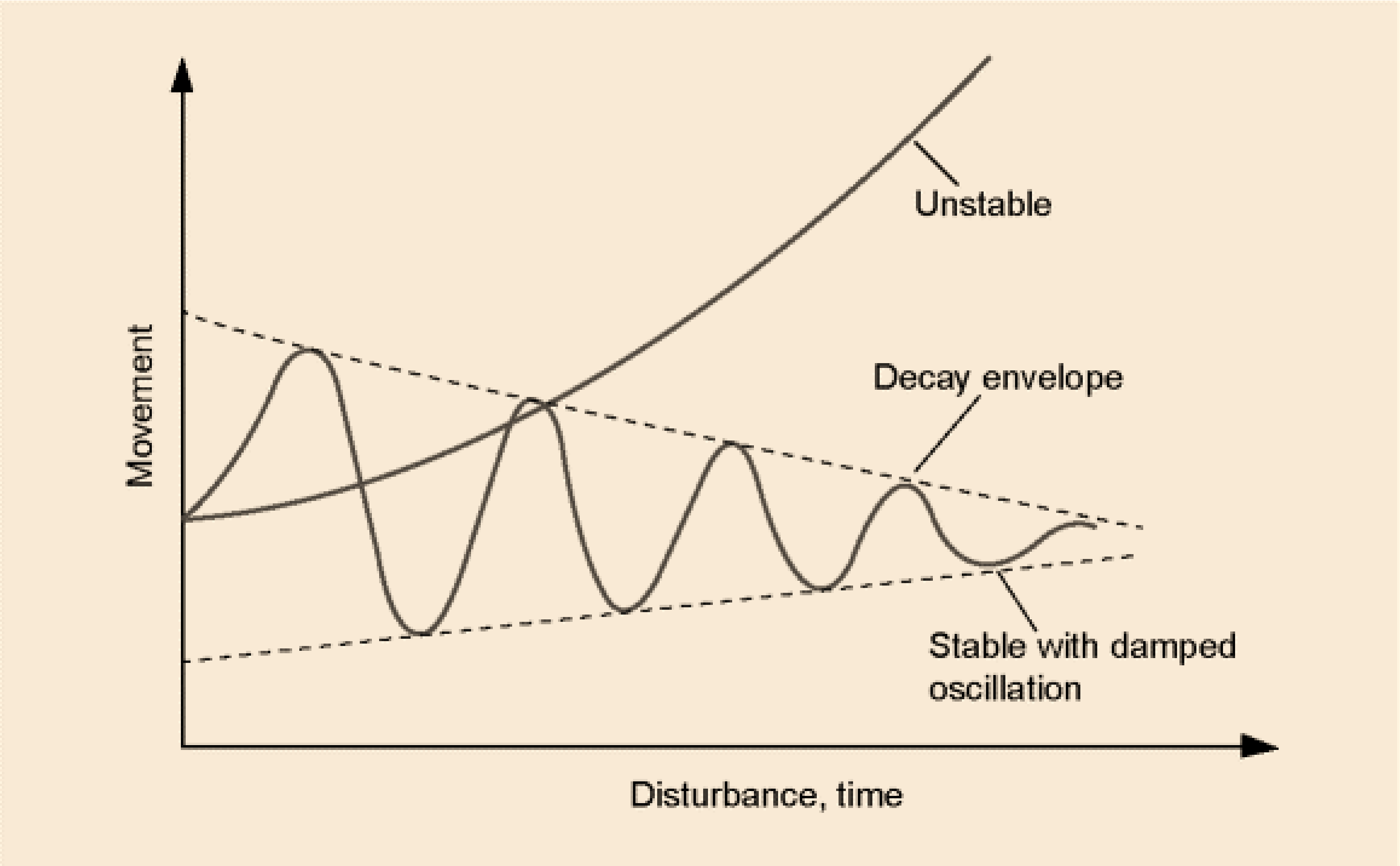
Definition and classification of power system stability

Power system stability may be broadly defined as a property of a power system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptance state of equilibrium after being subjected to a disturbance.



Definition and classification of power system stability

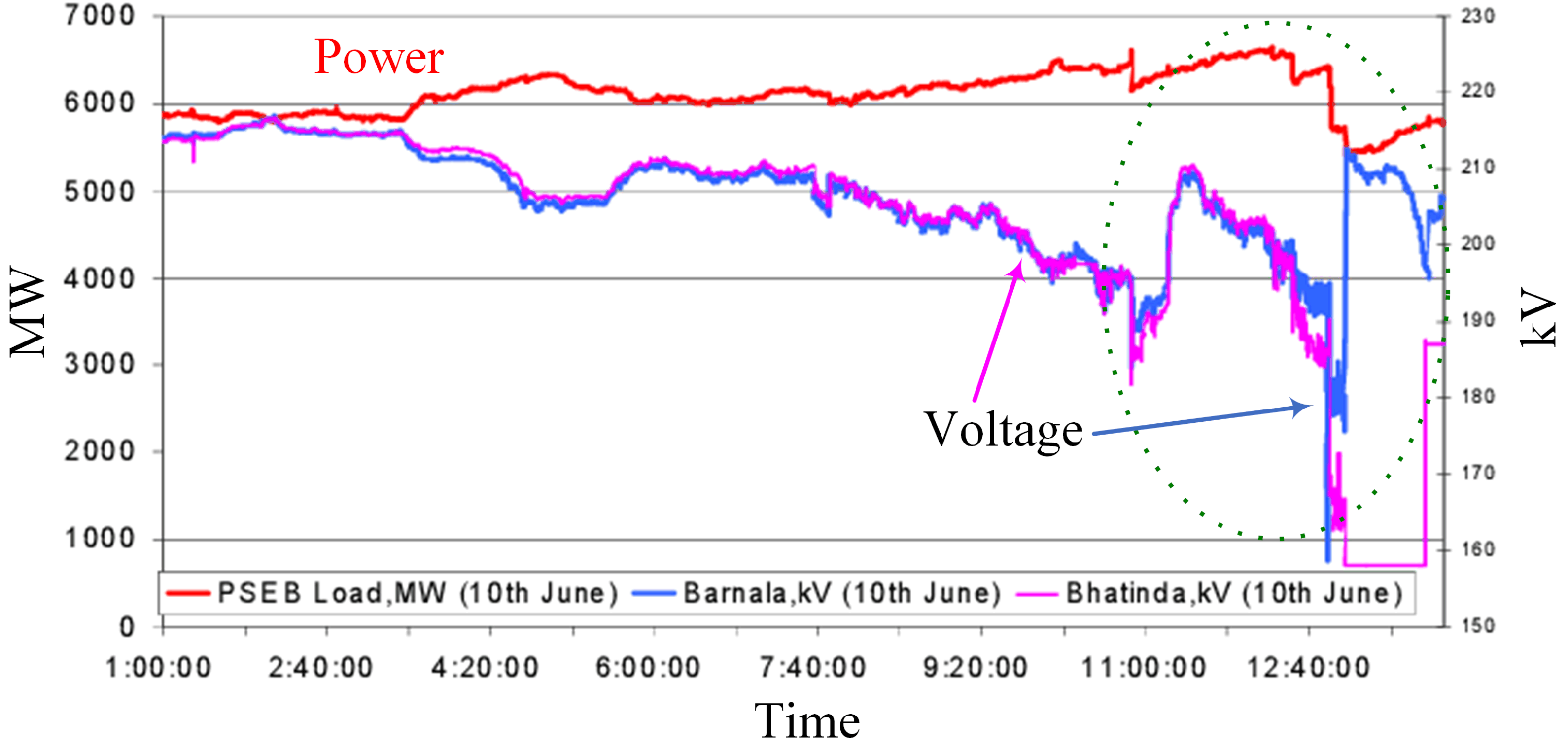
Rotor angle stability refers to the ability of synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance.



Definition and classification of power system stability

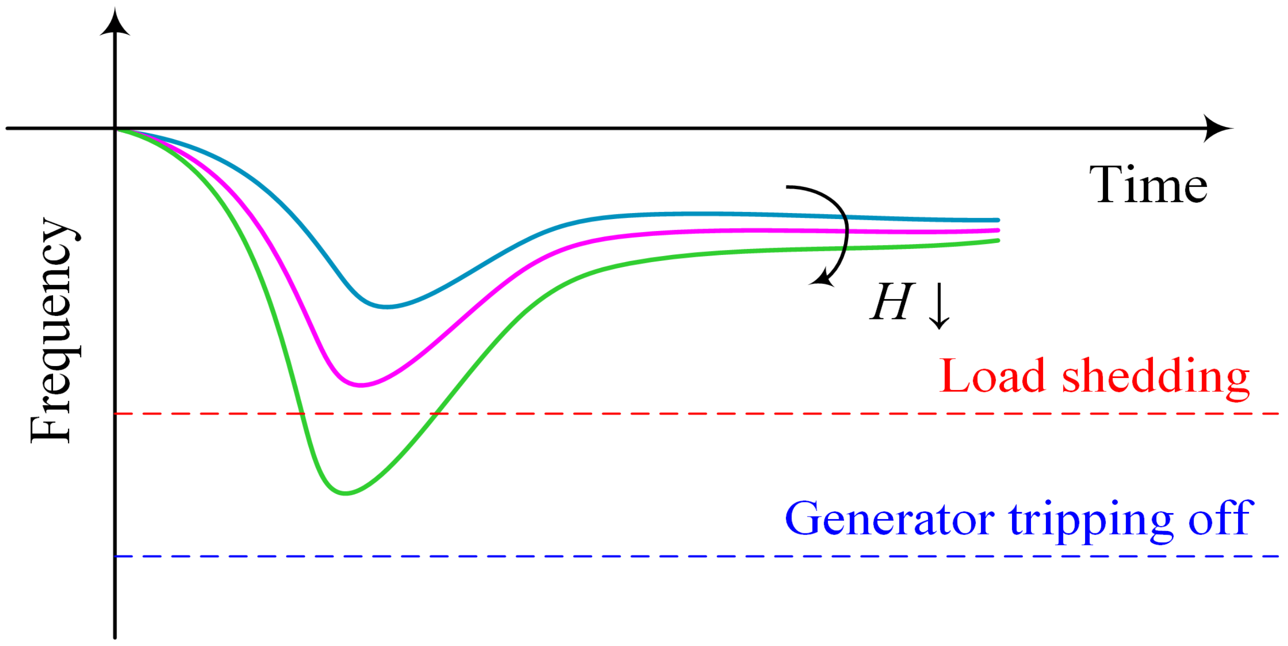
Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition.

Voltage Collapse at Punjab, India on June 10, 2007



Definition and classification of power system stability

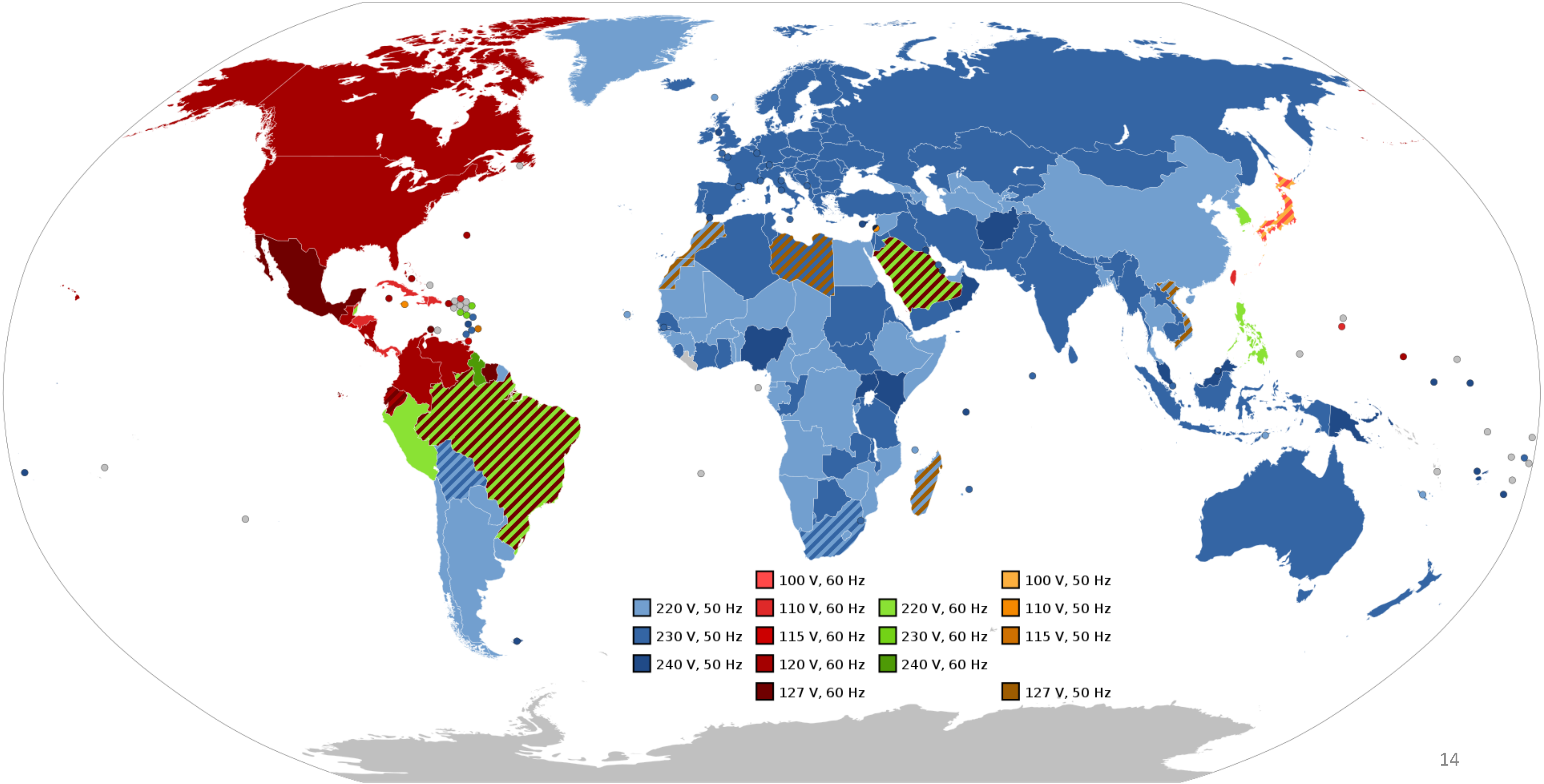
Frequency stability refers to the ability of a power system to maintain steady frequency following a severe system upset resulting in a significant imbalance between generation and load.



Hazards of underfrequency operation:

1) Operation of steam turbines below 58.5 Hz is severely restricted.
2) Output of plant auxiliaries (e.g., pumps, fans) may be significantly reduced.

Nominal frequency and voltage by country



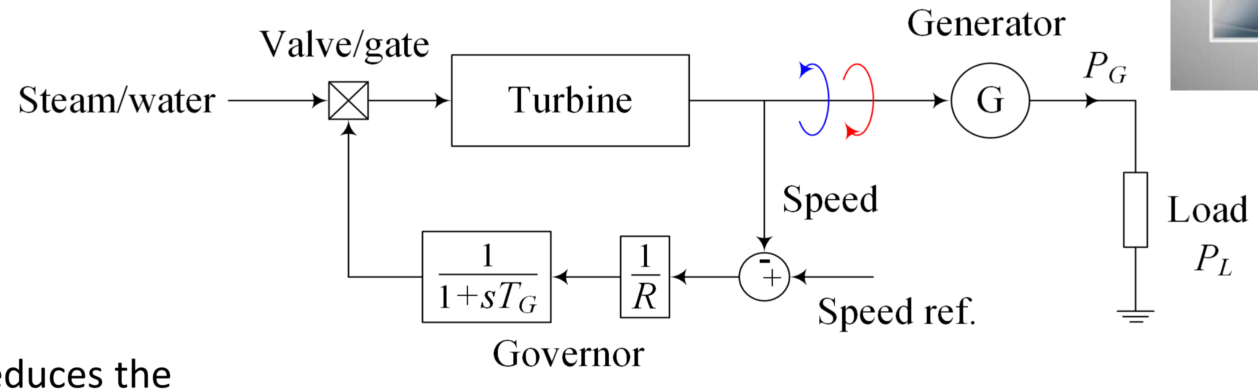
Frequency stability

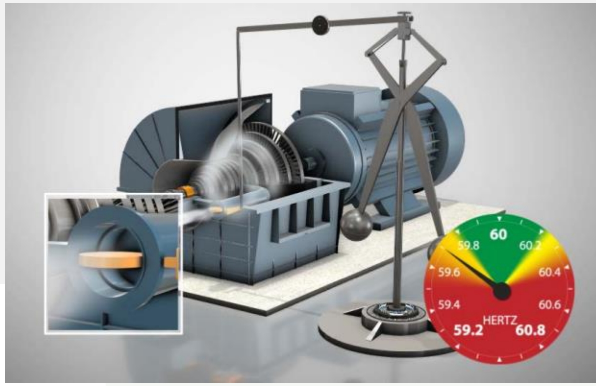
Automatically balance

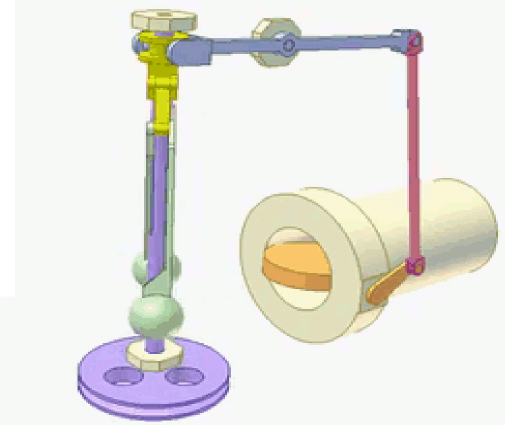
the supply and load power.

Grid frequency is regulated.

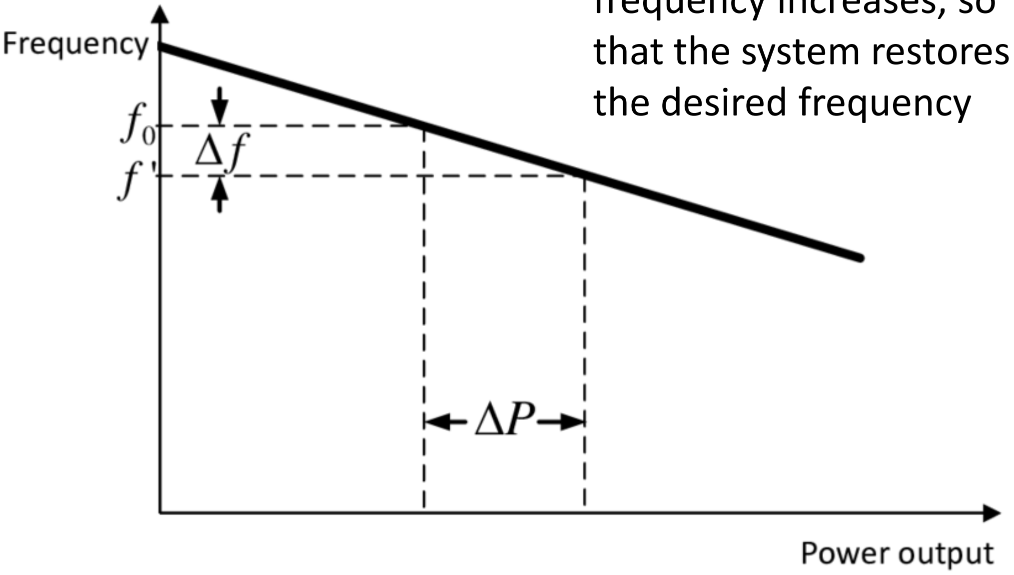
Governors with speed-droop characteristic

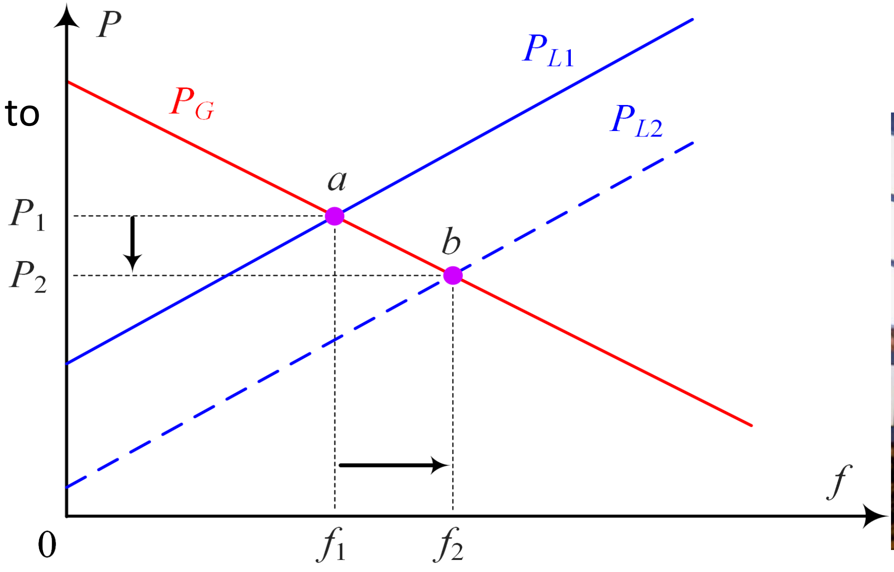




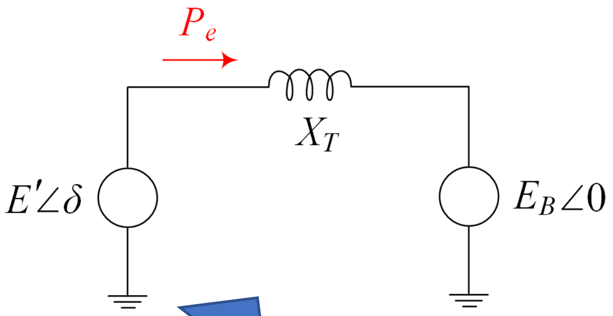


Governor reduces the generator power when frequency increases, so that the system restores to the desired frequency



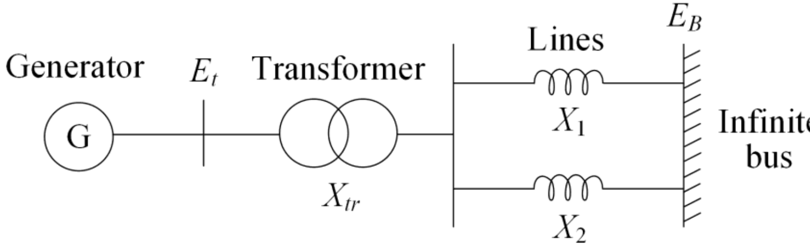


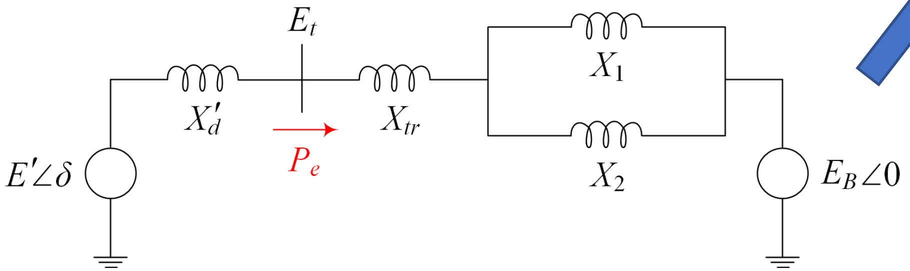
Small-disturbance rotor angle stability



$$
\begin{cases}
\dfrac{\mathrm{d}\delta}{\mathrm{d}t} = \omega - \omega_0 \\[2ex]
\dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \dfrac{\omega_0}{2H}\left(P_m - P_{max}\sin\delta - D(\omega - \omega_0)\right)
\end{cases}
$$

all in p.u. values



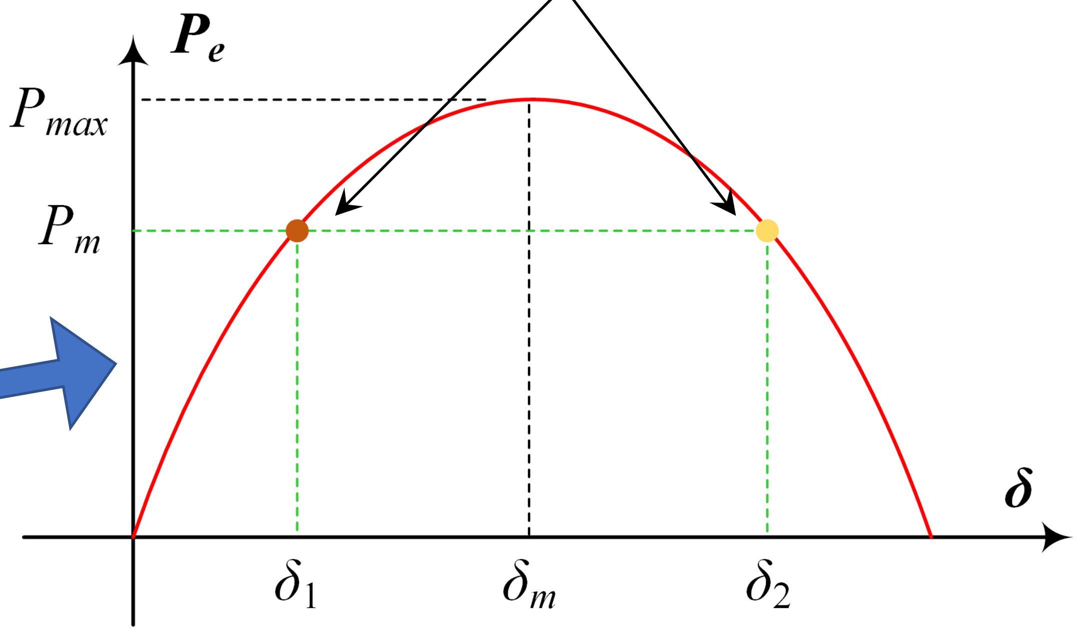


$$X_T = X_d^{'} + X_{tr} + X_1 // X_2$$

At equilibrium, $\dfrac{d\delta}{dt} = \dfrac{d\omega}{dt} = 0$

$$\Rightarrow \omega = \omega_0$$

$$P_m \approx P_{\max}\sin\delta = P_e$$



The generator’s electrical power output

$$P_e = \frac{E^{'}E_B}{X_T}\sin\delta \triangleq P_{max}\sin\delta$$

Power and Torque

- The real and reactive electric output power can be expressed in phase quantities as

$$P_{out} = 3V_\phi I_A \cos\theta$$

$$Q_{out} = 3V_\phi I_A \sin\theta$$

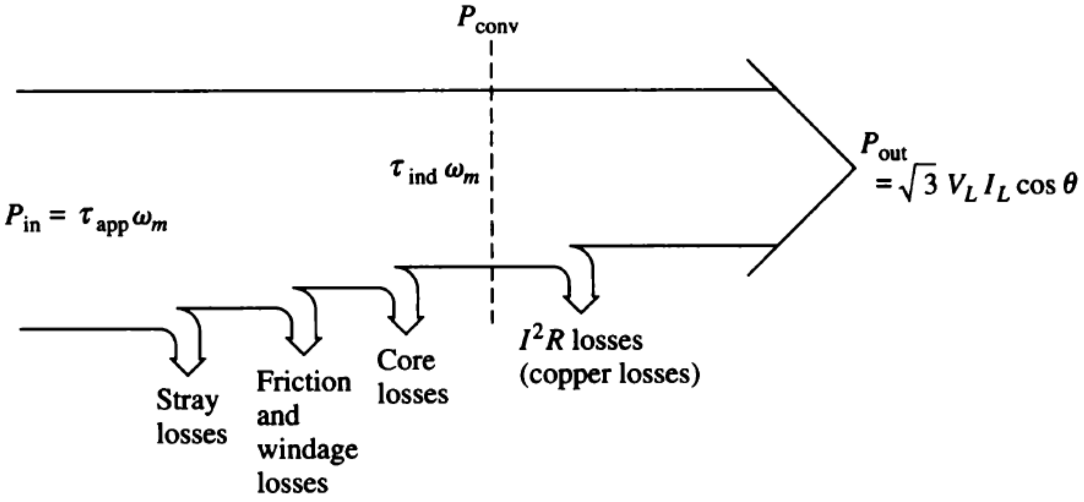
- Since

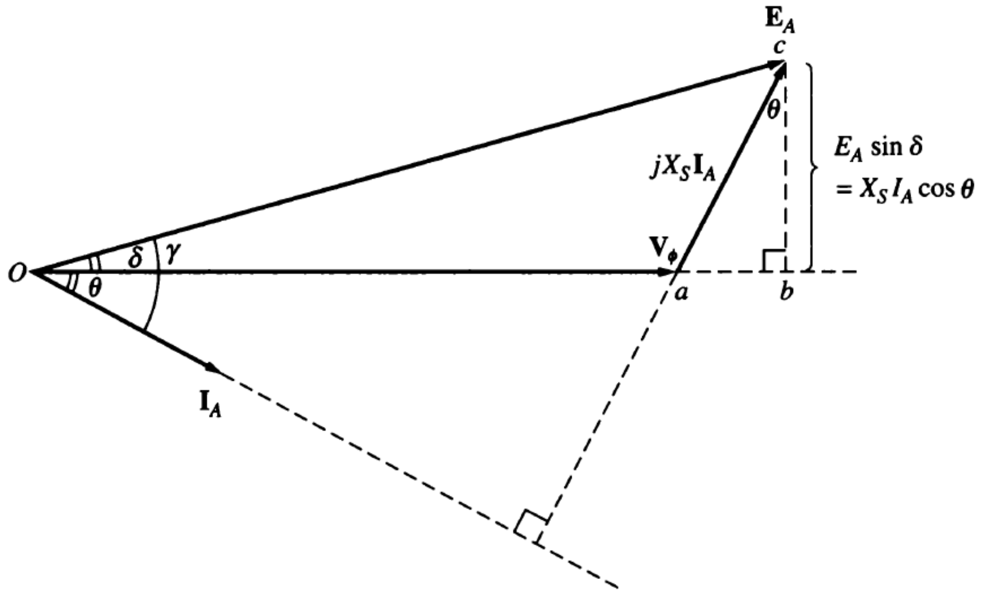
$$I_A \cos\theta = \frac{E_A \sin\delta}{X_S}$$

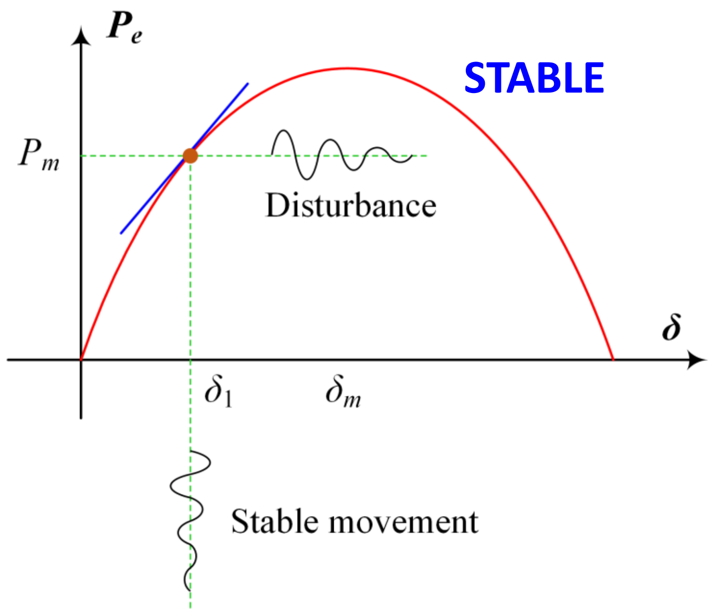
$$P_{out} = \tau\omega_m$$

- Therefore

$$P_{out} = \frac{3V_\phi E_A \sin\delta}{X_S} \qquad \tau = \frac{3V_\phi E_A \sin\delta}{X_S \omega_m}$$







Scenario 1: $\delta < \delta_1 \Rightarrow P_e < P_m \Rightarrow \frac{\mathrm{d}\omega}{\mathrm{d}t} > 0 \Rightarrow \omega > \omega_0$

(left side of δ_1)

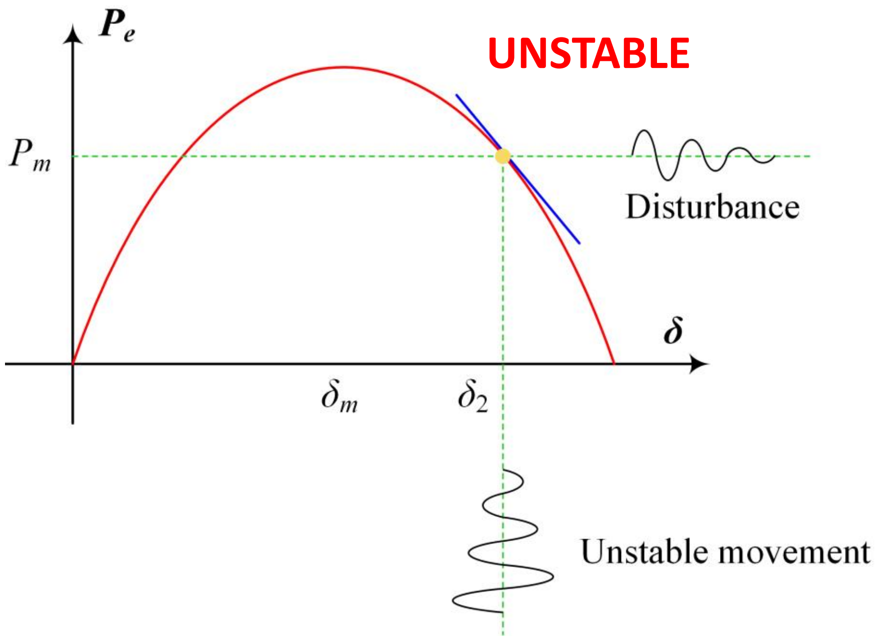
$\Rightarrow \delta \uparrow \Rightarrow \delta \to \delta_1$ return to original value

Scenario 2: $\delta_m > \delta > \delta_1 \Rightarrow P_e > P_m \Rightarrow \frac{\mathrm{d}\omega}{\mathrm{d}t} < 0$

(right side of δ_1)

$\Rightarrow \omega < \omega_0 \Rightarrow \delta \downarrow \Rightarrow \delta \to \delta_1$

return to original value



Scenario 3: $\delta < \delta_2 \Rightarrow P_e > P_m \Rightarrow \frac{\mathrm{d}\omega}{\mathrm{d}t} < 0$

(left side of δ_2)

$\Rightarrow \omega < \omega_0 \Rightarrow \delta \downarrow \Rightarrow \delta \ll \delta_2$

moving away from original value

Scenario 4: $\delta > \delta_2 \Rightarrow P_e < P_m \Rightarrow \frac{\mathrm{d}\omega}{\mathrm{d}t} > 0$

(right side of δ_2)

$\Rightarrow \omega > \omega_0 \Rightarrow \delta \uparrow \Rightarrow \delta \gg \delta_2$

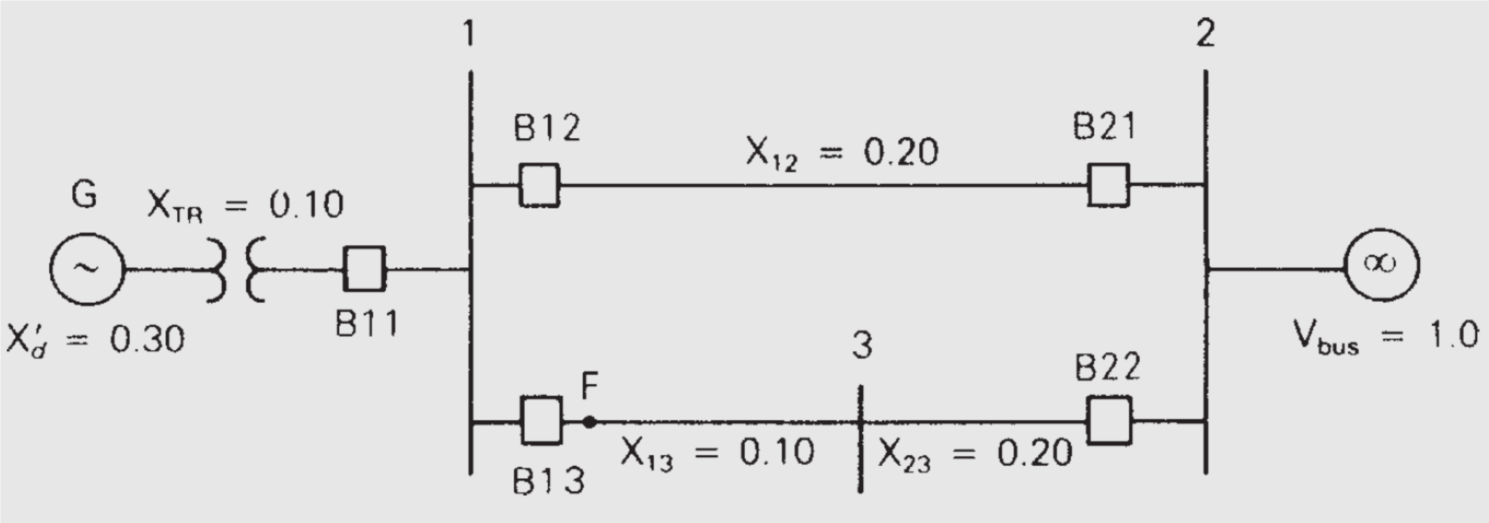
moving away from original value

Example: Generator internal voltage and real power output

The below figure shows a single-line diagram of a three-phase, 60-Hz synchronous generator, connected through a transformer and parallel transmission lines to an infinite bus. All reactances are given in per-unit on a common system base. If the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging, determine

(a) the internal voltage of the generator and

(b) the equation for the electrical power delivered by the generator versus its power angle δ.



Example: Generator internal voltage and real power output

Solution:

The equivalent circuit is shown below, from which the equivalent reactance between the machine internal voltage and infinite bus is

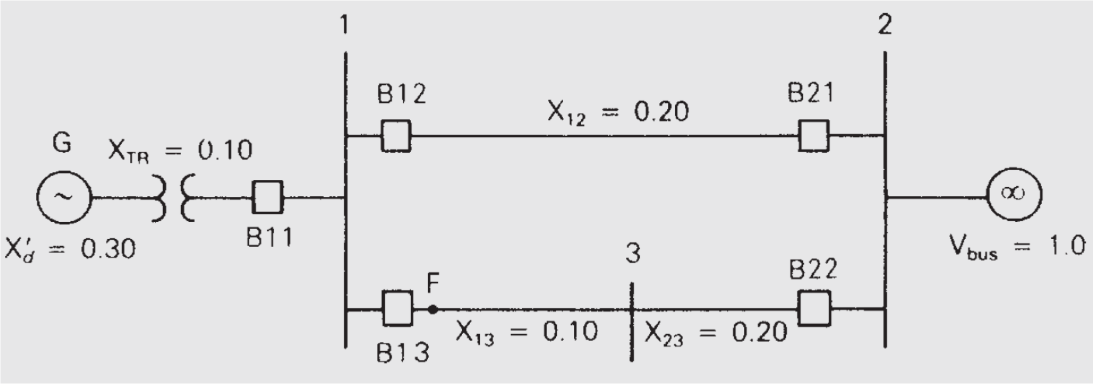
$$X_{eq} = X'_d + X_{TR} + X_{12} \| (X_{13} + X_{23})$$
$$= 0.30 + 0.10 + 0.20 \| (0.10 + 0.20)$$
$$= 0.520 \quad \text{per unit}$$

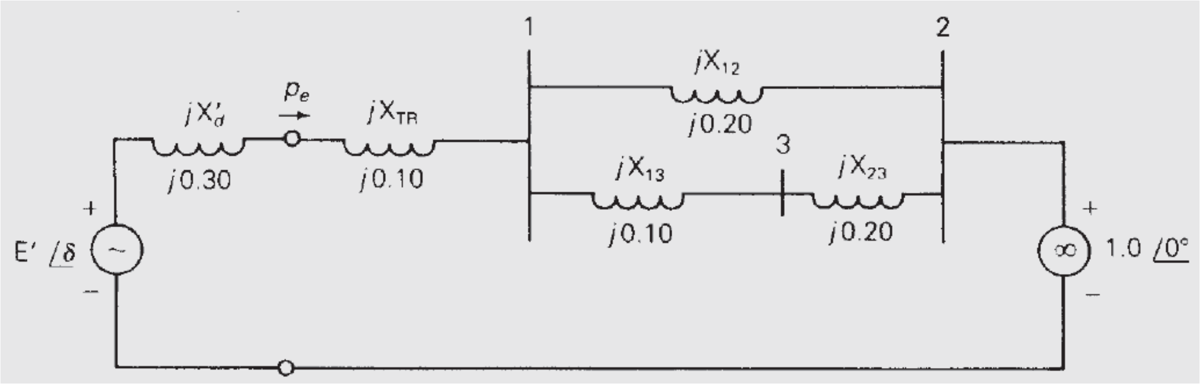
The current into the infinite bus is

$$I = \frac{P}{V_{bus}(\text{p.f.})} \angle -\cos^{-1}(\text{p.f.}) = \frac{(1.0)}{(1.0)(0.95)} \angle -\cos^{-1} 0.95$$
$$= 1.05263 \angle -18.195^\circ \quad \text{per unit}$$

$$E' = E' \angle \delta = V_{bus} + jX_{eq} I$$
$$= 1.0 \angle 0^\circ + (j0.520)(1.05263 \angle -18.195^\circ)$$
$$= 1.0 \angle 0^\circ + 0.54737 \angle 71.805^\circ$$
$$= 1.1709 + j0.5200$$
$$= 1.2812 \angle 23.946^\circ \quad \text{per unit}$$

$$p_e = \frac{(1.2812)(1.0)}{0.520} \sin \delta = 2.4628 \sin \delta \quad \text{per unit}$$





Stability under sudden change of mechanical power

The equation of motion is

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = \frac{\omega_0}{2H}(P_m - P_e), \quad P_e = P_{max}\sin\delta$$

This is Newton's law, which means that the machine accelerates if $P_m > P_e$.

If we multiply the equation by the speed, we will get the energy equation:

$$2\frac{\mathrm{d}\delta}{\mathrm{d}t}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = \frac{\omega_0}{H}(P_m - P_e)\frac{\mathrm{d}\delta}{\mathrm{d}t}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\mathrm{d}\delta}{\mathrm{d}t}\right]^2 = \frac{\omega_0}{H}(P_m - P_e)\frac{\mathrm{d}\delta}{\mathrm{d}t}$$

$$\int \mathrm{d}\left[\frac{\mathrm{d}\delta}{\mathrm{d}t}\right]^2 = \int \frac{\omega_0}{H}(P_m - P_e)\mathrm{d}\delta \qquad (*)$$

Suppose at the beginning, the machine is stable, with P_{m0} and δ_0 in equilibrium.

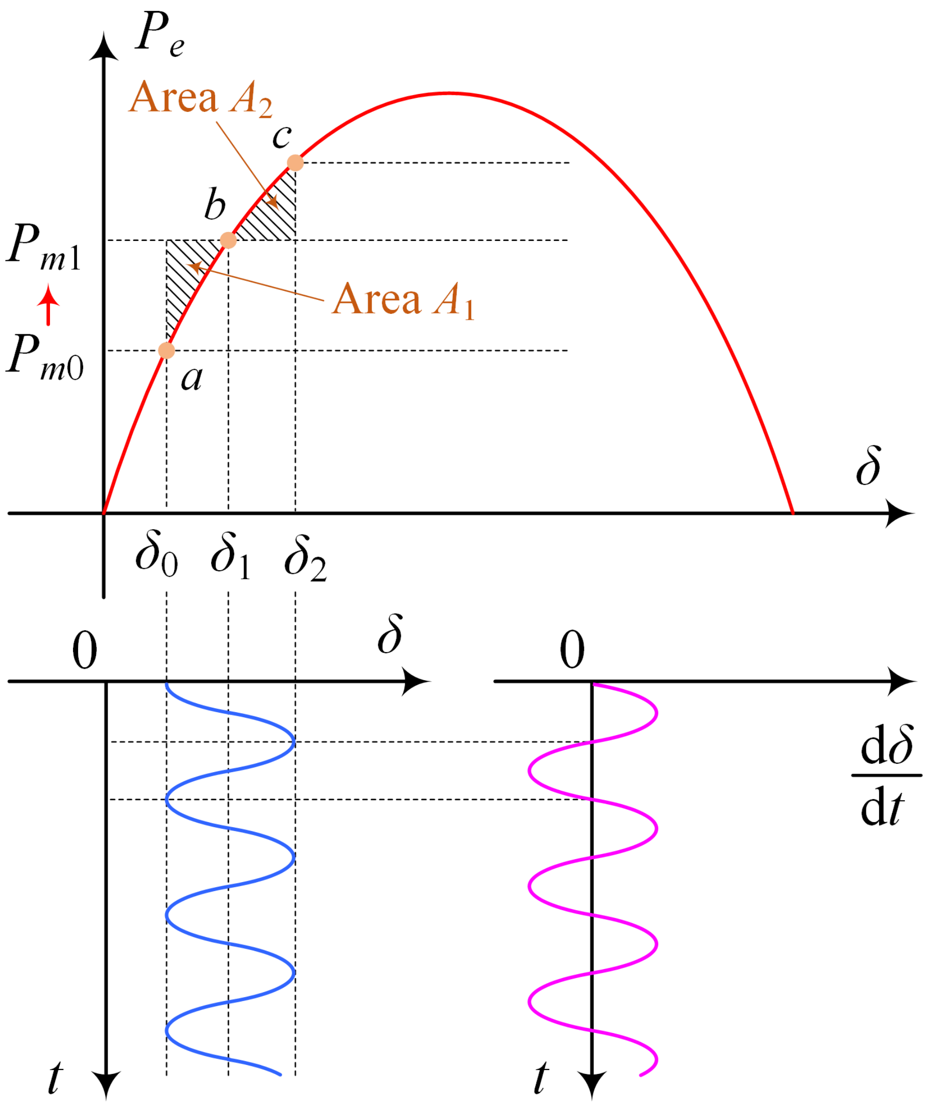
Now, if P_m suddenly increases to P_{m1}, the machine will accelerate, causing δ to increase, and hence P_e also increases.

Then, soon, P_e equals P_{m1} , and acceleration = 0. At this point (say $\delta = \delta_1$), δ will still increase because the speed is not zero though acceleration is zero.

The machine will decelerate (slow down) because P_e now exceeds $P_{m1,}$ according to the Newton law equation. It will soon stop (i.e., dδ/dt reduces to 0). At this point, say $\delta = \delta_2$, the angle δ moves back. If there is some damping, it swings back and forth between δ_0 and $\delta_{2.}$, with reducing amplitude and eventually converges back to δ_1, which is the new equilibrium.

The question is: would this ever be stable?

Equal-area criterion



converges to 0 if there is some damping

Energy equation

$$\int d\left[\frac{d\delta}{dt}\right]^2 = \int \frac{\omega_0}{H}(P_m - P_e)d\delta$$

From δ_0 to δ_1, to δ_2, energy changes accordingly, and **the system moves and must gain no energy in order not to fly away!**

$$\int_{\delta_0}^{\delta_1} (P_m - P_e)d\delta = \text{Area } A_1$$

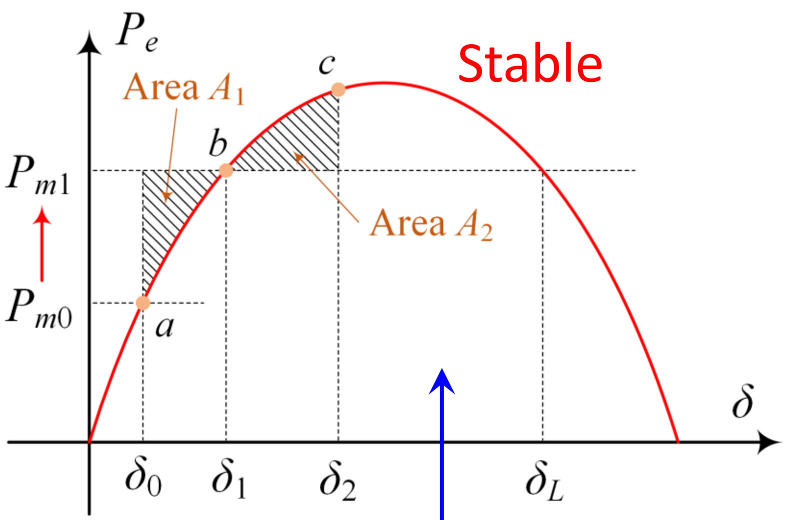
$$\int_{\delta_1}^{\delta_2} (P_m - P_e)d\delta = \text{Area } A_2$$

Clearly, the acceleration energy should equal the deceleration energy in order to be stable.

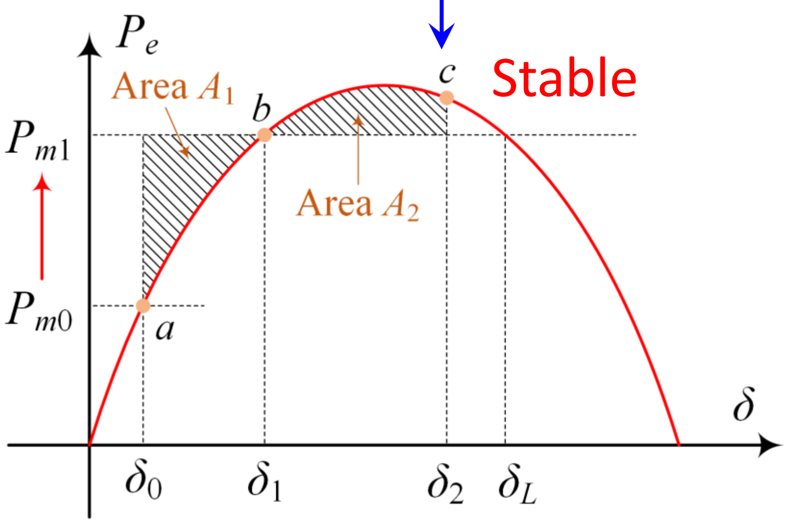
$$A_1 = A_2$$

Thus, $$\delta_2 < \pi - \delta_0$$

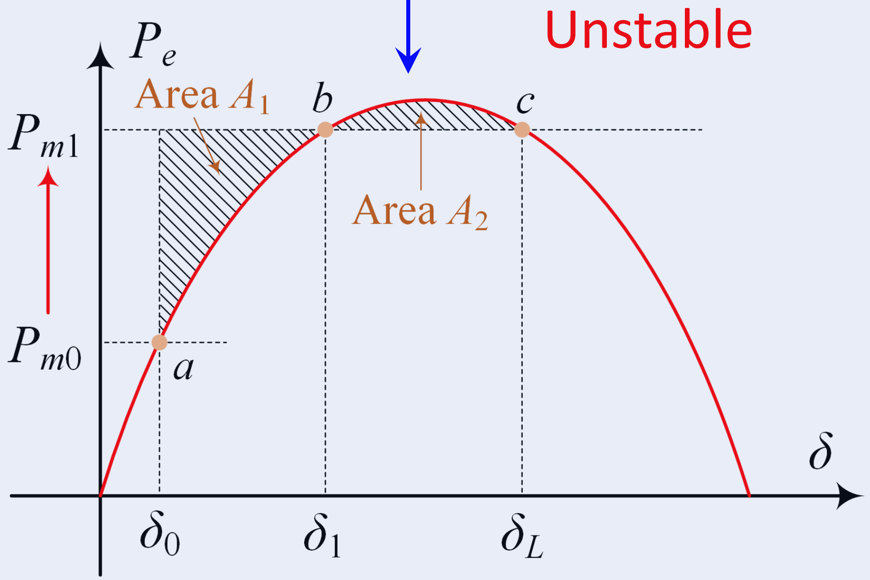
Equal-area criterion (EAC)



Area A_1 = Area A_2



Area A_1 > Area A_2

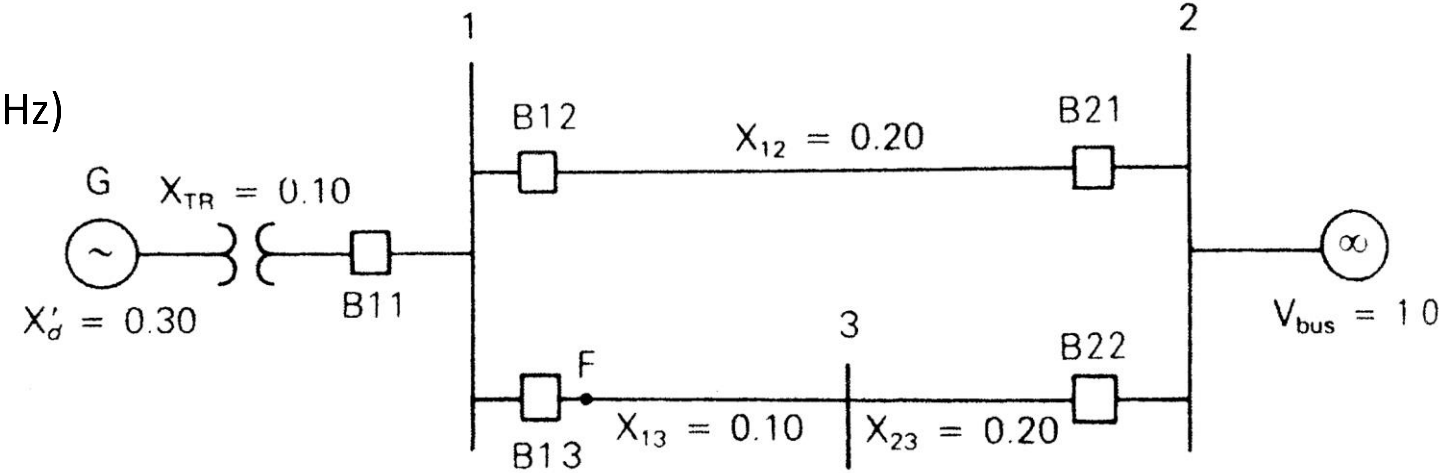


Excess energy after returning to δ_1

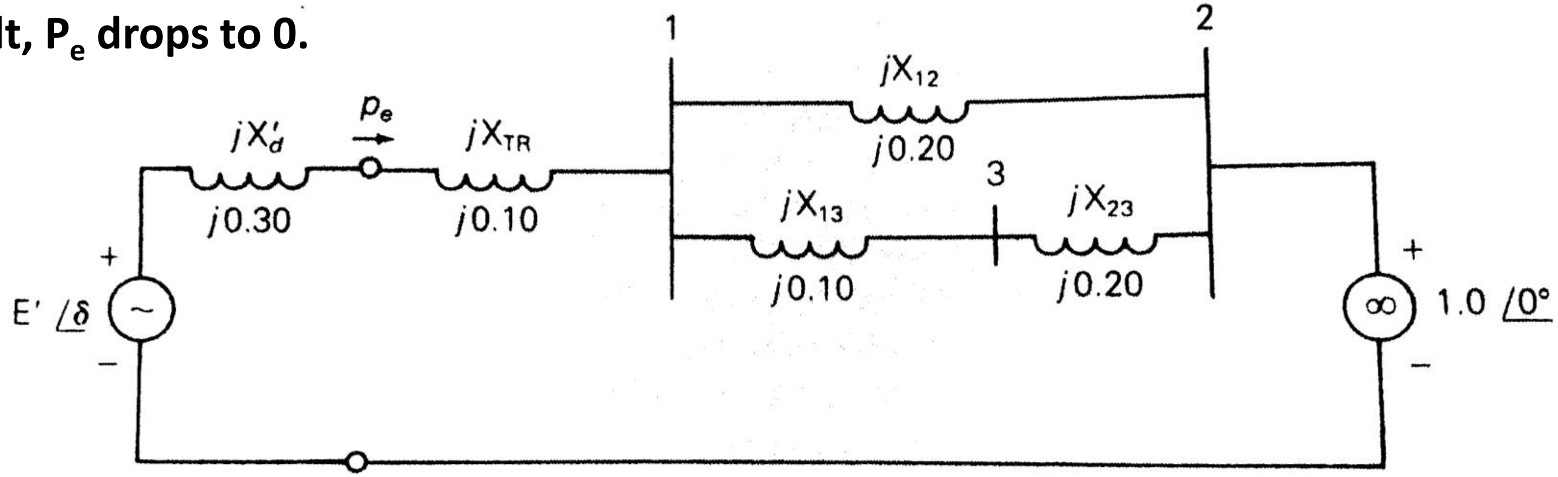
System expands → Unstable

Example 1: three-phase fault

Suppose a short-circuit fault occurs at F.
Fault cleared after 3 cycles (i.e., 0.05 s at 60 Hz)
All circuit breakers remain closed.
Assume P_m remains constant throughout the disturbance.
Inertia constant H = 3 pu-s.
Synchronous speed = 1 pu.



Start at δ_0 = 23.95° = 0.4179 rad, with P_m = 1 pu. From example on page 19.
At first, $P_e = P_m$ = 1 pu. **Instantly after fault, P_e drops to 0.**



$$\frac{2H}{\omega_s}\ddot{\delta}(t) = P_m - \underset{0}{\cancel{P_e}}$$

Integrating twice to get

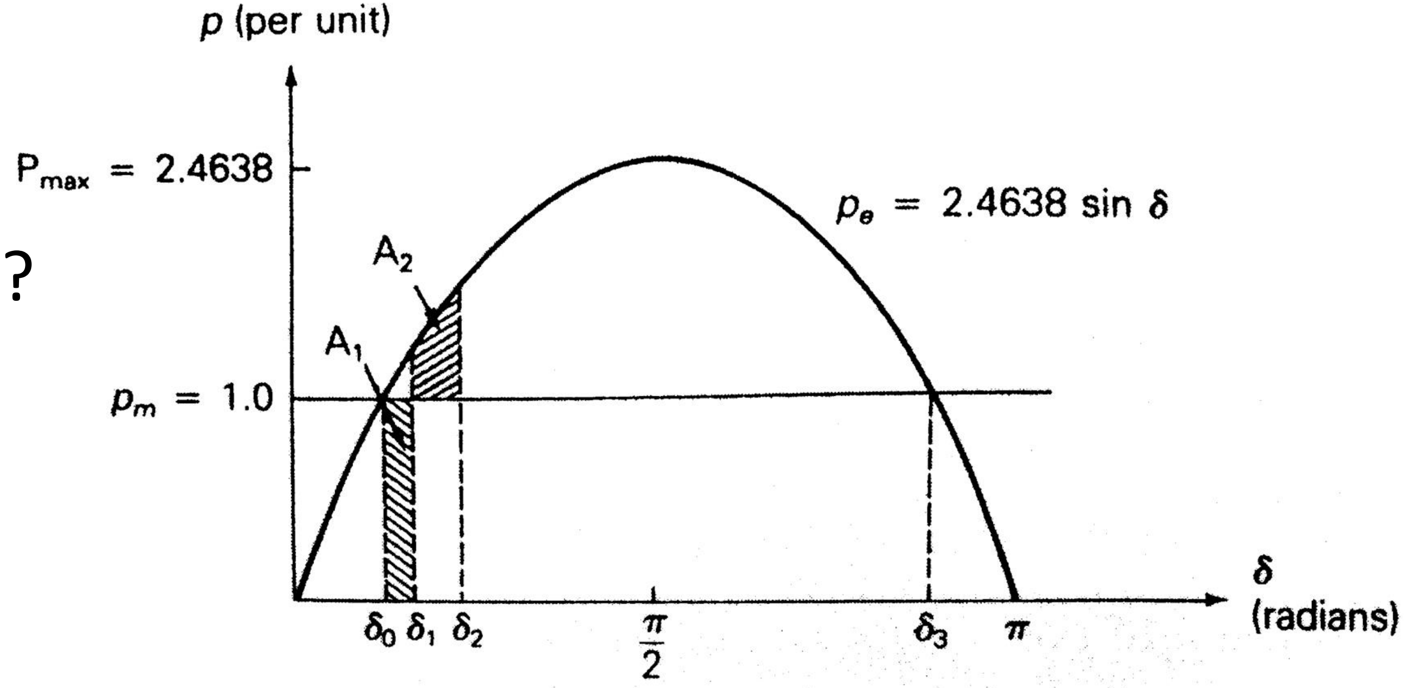
$$\dot{\delta}(t) = \frac{\omega_s P_m}{2H}t + 0$$

$$\delta(t) = \frac{\omega_0 P_m}{4H}t^2 + \delta_0$$ (see page 9)

At t = 0.05 s, δ_1 = (2π(60)/12) (0.05²) + 0.4179 rad = 0.4964 rad = 28.44°.

Example 1: three-phase fault

Can it be stable?
What's the max power angle?



Instantly after fault, P_e drops to 0.

The accelerating area A_1 $= \int_{\delta_0}^{\delta_1} P_m d\delta = \int_{\delta_0}^{\delta_1} 1.0 d\delta = 0.4964 - 0.4179 = 0.0785$

After fault clearance at 0.05s, P_e moves back to the curve at $\delta = \delta_1$. The curve is P_e = 2.4638 sinδ.
Then, δ increases but decelerates to zero, stopping at δ_2.

The decelerating area A_2 $= \int_{\delta_1}^{\delta_2} (P_e - P_m) d\delta = \int_{\delta_1}^{\delta_2} (2.4638 \sin\delta - 1.0) d\delta$

Example 1: three-phase fault

If the system is stable, **A1 must equal A2** so that energy does not expand.

$$0.0785 = \int_{\delta_1}^{\delta_2} (2.4638 \sin\delta - 1.0) d\delta$$

$$0.0785 = 2.4638[\cos(0.4964 \text{ rad}) - \cos\delta_2] - (\delta_2 - 0.4964)$$

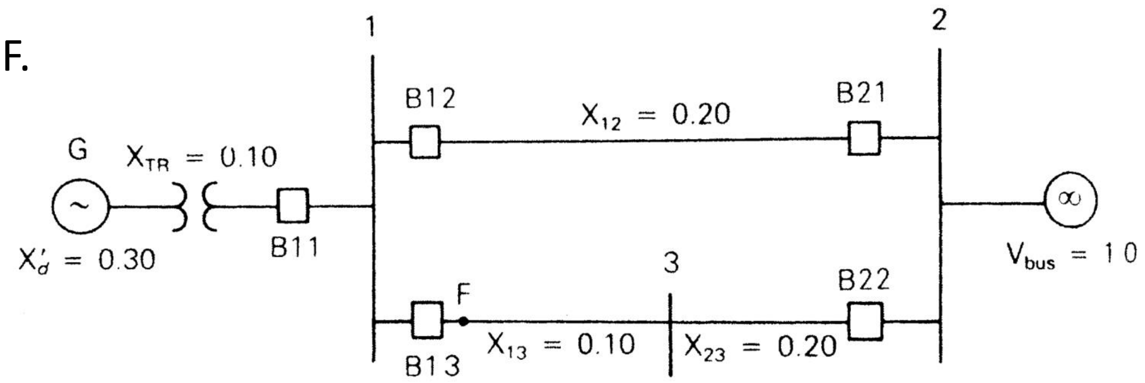
SOLUTION: $\delta_2 = 0.7003 \text{ rad} = 40.12^{\text{o}} \quad < 180^{\text{o}} - \delta_0????$

Clearly, δ_2 does not exceed 180º – δ_0 = 156.05º. The system is STABLE.

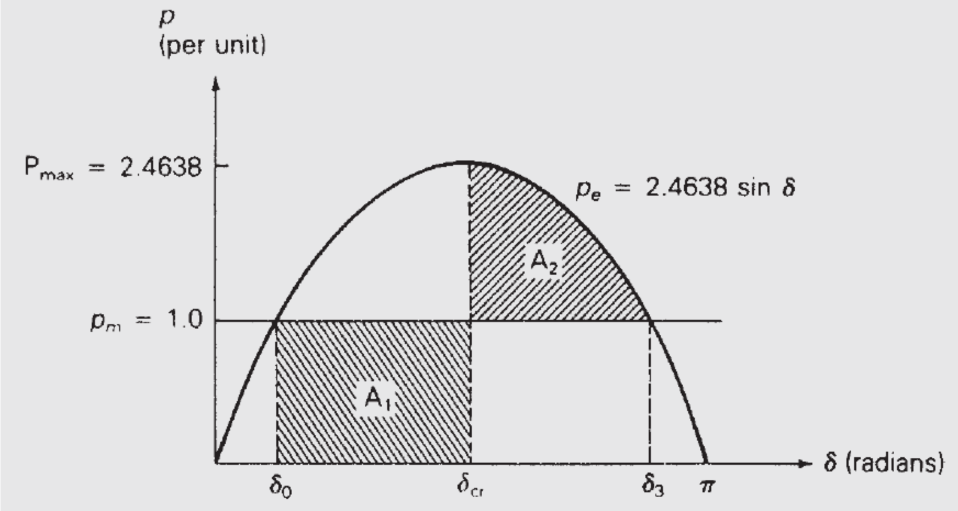
In practice, engineers have to figure out the safe clearing time! Here, we assume it's 3 cycles, i.e., 0.05 s, and found that it can remain stable. How about 4 cycles? Is it still stable? **What is the critical time?** Assignment!

Example 2: three-phase fault – critical clearing time

Suppose the same as Example 2: a short-circuit fault occurs at F.
But fault cleared NOT after 3 cycles. (60 Hz)
Determine the critical clearing time to ensure the stability.



Solution:
At the critical clearing angle, denoted δ_{cr}, the fault is extinguished. The power angle then increases to a maximum value δ_3 =180° - δ_0 =156.05° = 2.7236 radians, which gives the maximum decelerating area. Equating the accelerating and decelerating areas,



$$A_1 = \int_{\delta_0}^{\delta_{cr}} p_m \, d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{max} \sin\delta - p_m) d\delta$$

$$\int_{0.4179}^{\delta_{cr}} 1.0 \, d\delta = \int_{\delta_{cr}}^{2.7236} (2.4638 \sin\delta - 1.0) \, d\delta$$

Solving for δ_{cr},

$$(\delta_{cr} - 0.4179) = 2.4638[\cos\delta_{cr} - \cos(2.7236)] - (2.7236 - \delta_{cr})$$

$$2.4638 \cos\delta_{cr} = +0.05402$$

$$\delta_{cr} = 1.5489 \text{ radians} = 88.74°$$

$$\delta(t) = \frac{\omega_{syn} p_{mp.u.}}{4H} t^2 + \delta_0$$

Solving

$$t = \sqrt{\frac{4H}{\omega_{syn} p_{mp.u.}} (\delta(t) - \delta_0)}$$

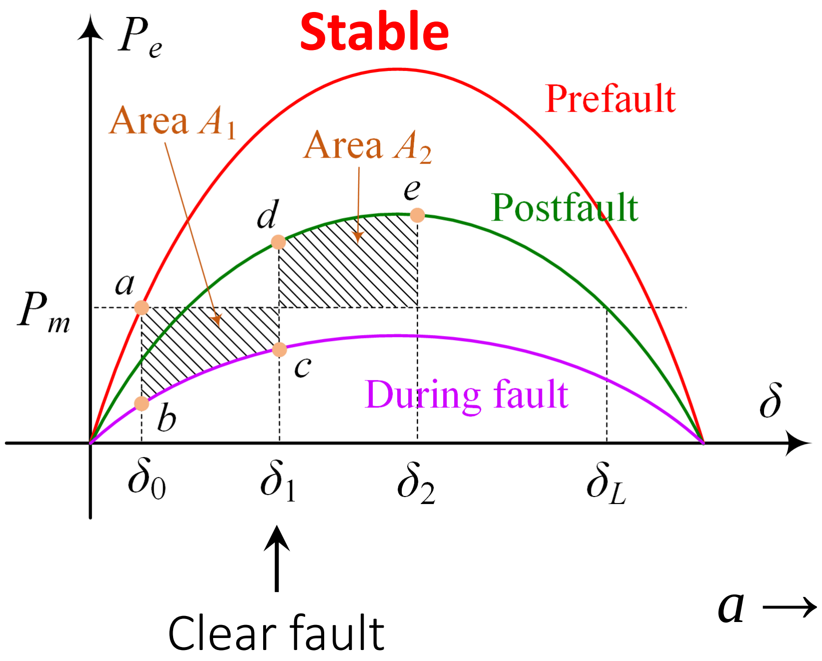
Using $\delta(t_{cr}) = \delta_{cr} = 1.5489$ and $\delta_0 = 0.4179$ radian,

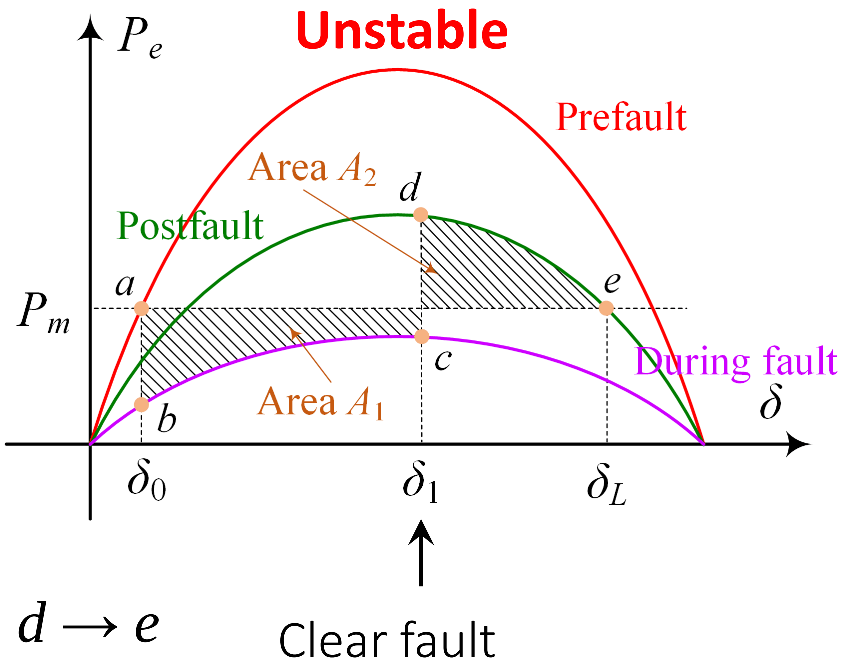
$$t_{cr} = \sqrt{\frac{12}{(2\pi 60)(1.0)} (1.5489 - 0.4179)}$$

$$= 0.1897 \text{ s} = 11.38 \text{ cycles}$$

More general case

In practice, P_e may fall to another power curve instead of ZERO power. The same EAC holds.





$$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$$

Area A_1 = Area A_2

Area A_1 > Area A_2

$$\int \mathrm{d}\left[\frac{\mathrm{d}\delta}{\mathrm{d}t}\right]^2 = \int \frac{\omega_0}{H}(P_m - P_e)\mathrm{d}\delta$$

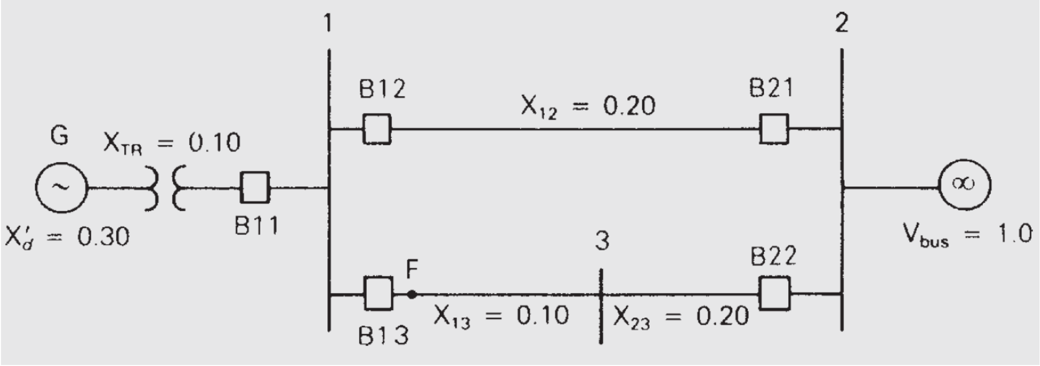
Example 3: electrical power curve changed

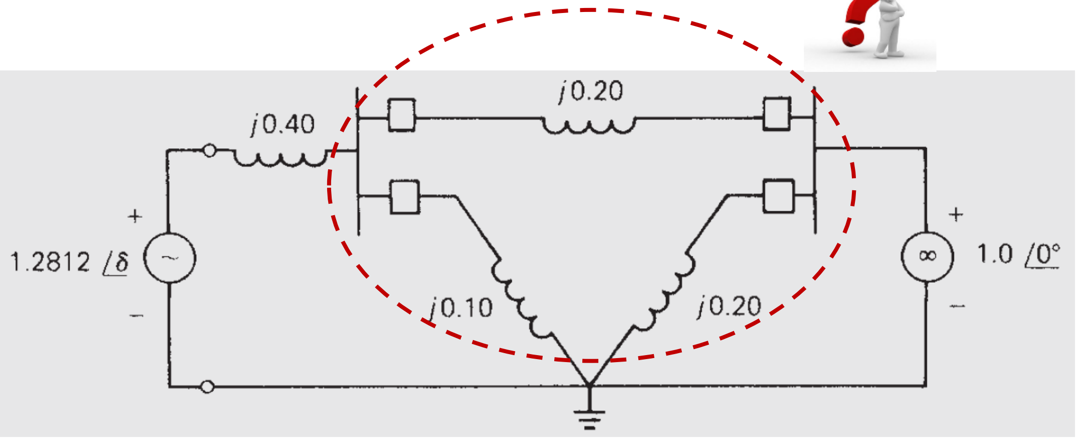
The below figure shows a single-line diagram of a three-phase, 60-Hz synchronous generator, connected through a transformer and parallel transmission lines to an infinite bus. All reactances are given in per-unit on a common system base. If the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging, determine

(a) the internal voltage of the generator and

(b) the equation for the electrical power delivered by the generator versus its power angle δ.

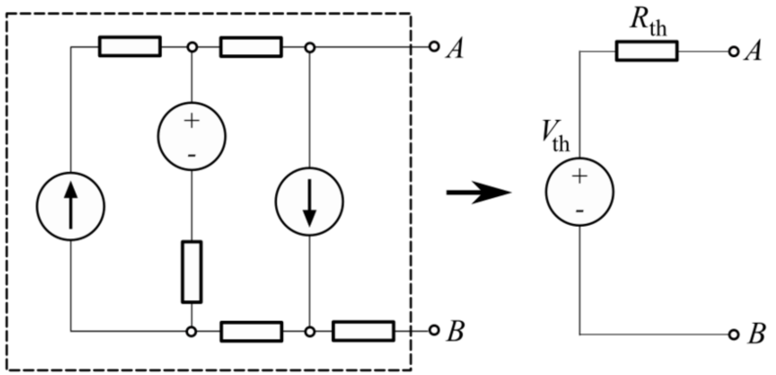
But when a permanent three-phase-to-ground bolted short circuit occurs on line 1–3 at bus 3. The fault is cleared by opening the circuit breakers at the ends of line 1–3 and line 2–3. These circuit breakers then remain open. Calculate the critical clearing angle.

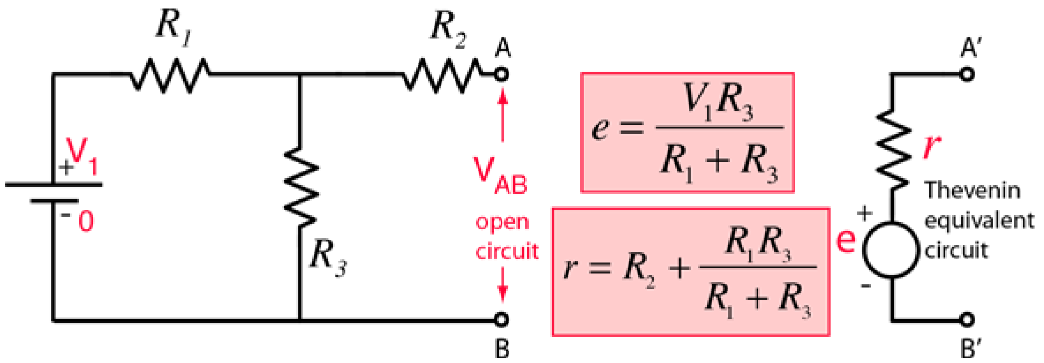




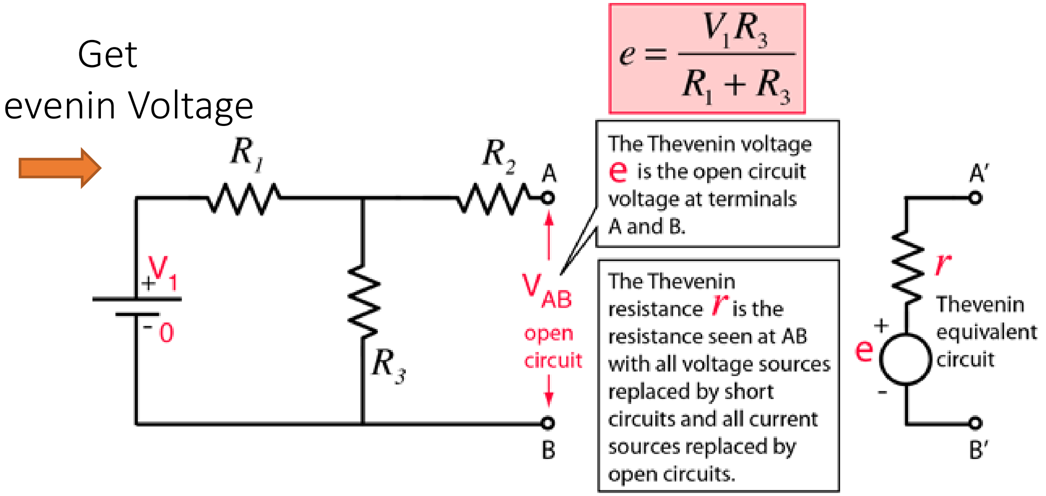
Thevenin's theorem in power systems

Thevenin theorem is an analytical method used to change a complex circuit into a simple equivalent circuit consisting of a single resistance in series with a source voltage.

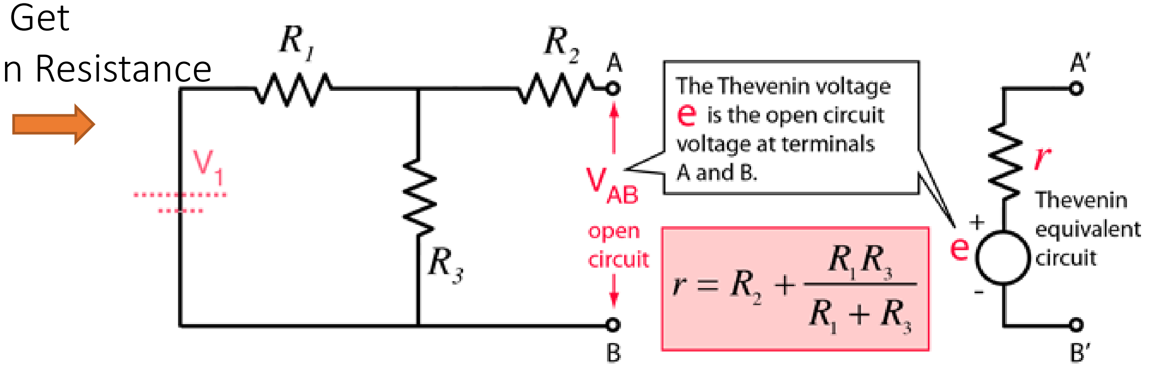




Get
Thevenin Voltage



Get
Thevenin Resistance

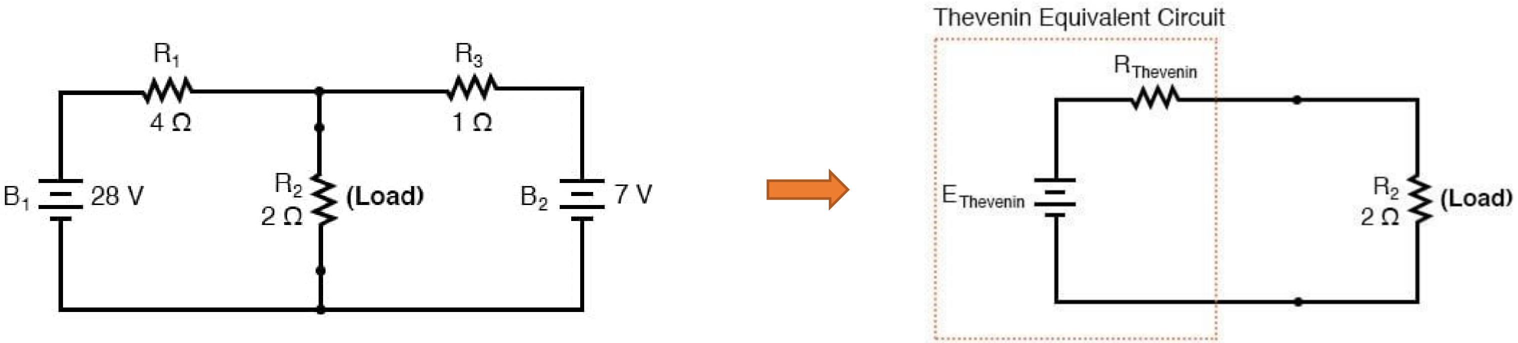


Experimentally the Thevenin resistance can be found by progressively loading the circuit until its output voltage drops to half the open circuit voltage. At that point the load resistance is equal to the Thevenin resistance.

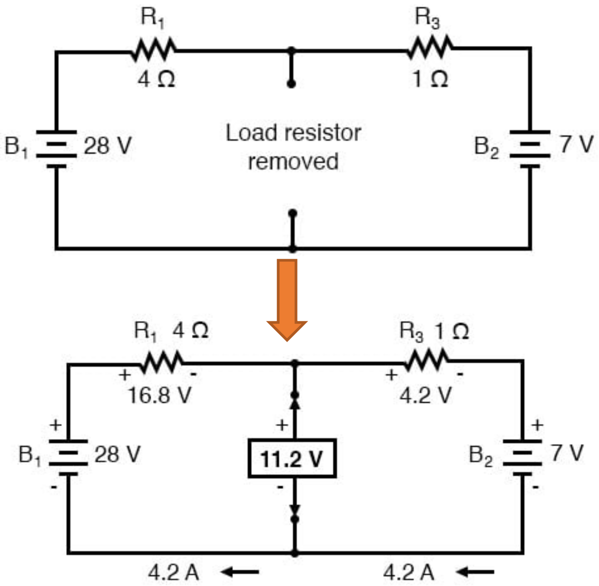
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/thevenin.html#c2

Example: Thevenin equivalent circuit

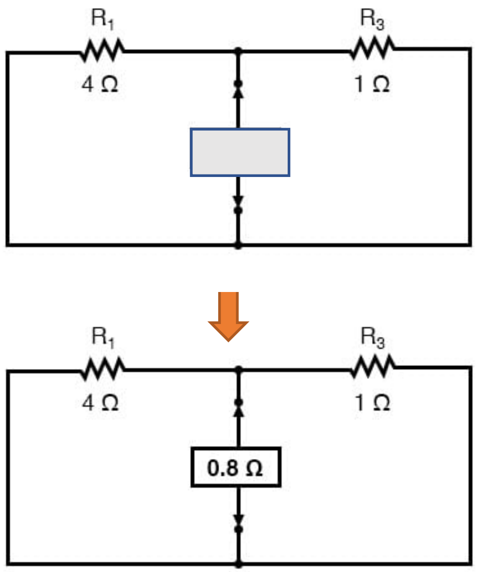
The load resistance can then be re-connected to this "Thevenin equivalent circuit" and calculations carried out as if the whole network were nothing but a simple series circuit:

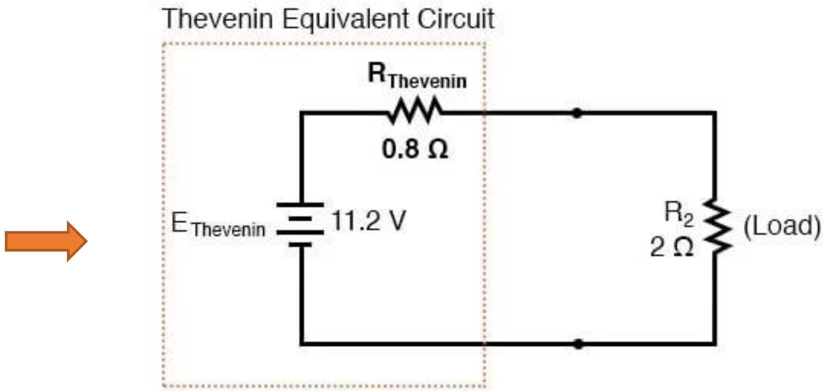


1. Get Thevenin Voltage:
Open circuit



2. Get Thevenin Voltage:
Short circuit





Example 3: electrical power curve changed

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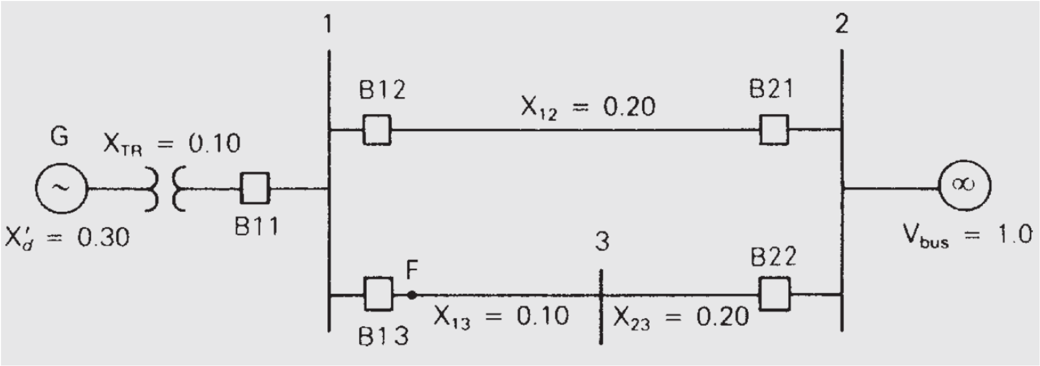
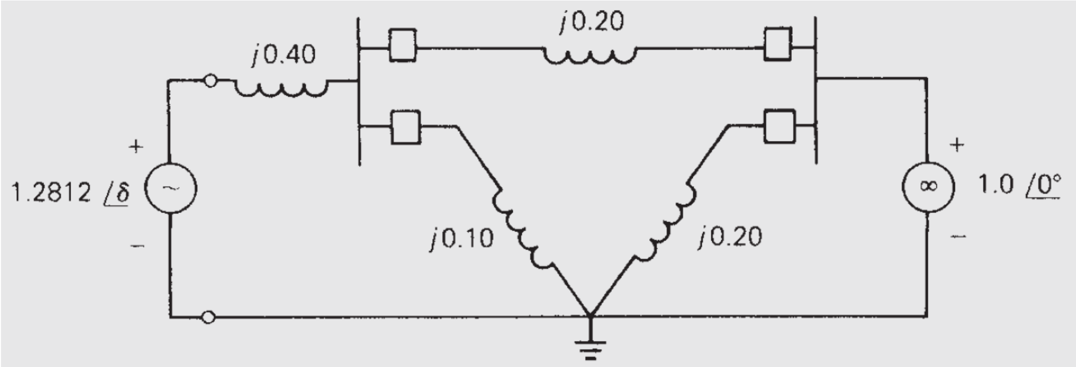
But when a permanent three-phase-to-ground bolted short circuit occurs on line 1–3 at bus 3. The fault is cleared by opening the circuit breakers at the ends of line 1–3 and line 2–3. These circuit breakers then remain open. Calculate the critical clearing angle.

As in previous examples, H=3.0 p.u.-s, p_m=1.0 per unit and $\omega_{p.u}$=1.0 in the swing equation.





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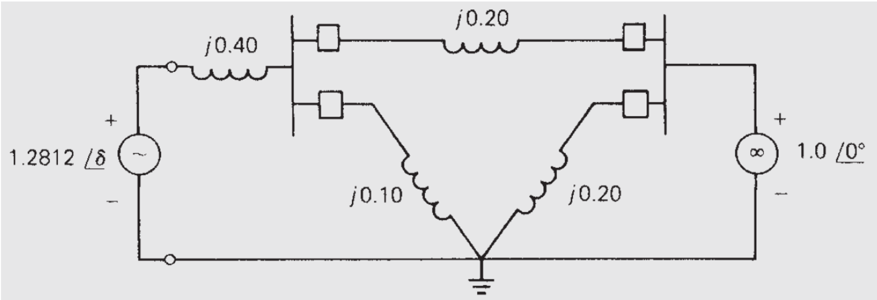
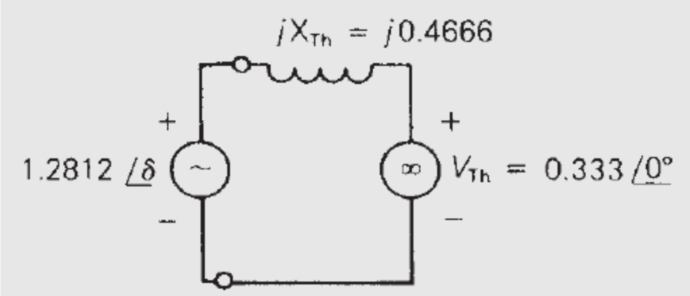
As in previous examples, H=3.0 p.u.-s, p_m=1.0 per unit and $\omega_{p.u}$=1.0 in the swing equation.

Solution:

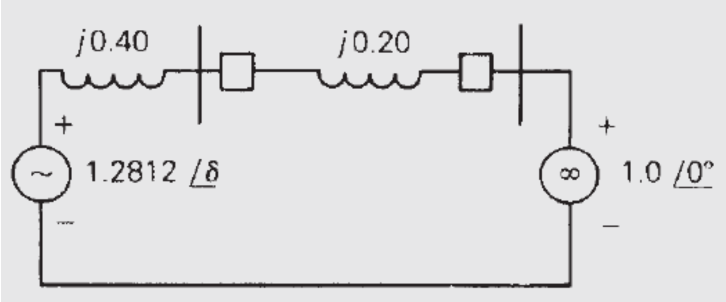
Via Thevenin equivalent to simplify the ground connected circuit



Faulted network



Thevenin equivalent of faulted network



Postfault network

Thevenin voltage and reactance

$$X_{Th} = 0.40 + 0.20 \| 0.10 = 0.46666 \quad \text{per unit}$$

$$V_{Th} = 1.0\underline{/0^\circ}\left[\frac{X_{13}}{X_{13}+X_{12}}\right] = 1.0\underline{/0^\circ}\,\frac{0.10}{0.30}$$
$$= 0.33333\underline{/0^\circ} \quad \text{per unit}$$

the equation for the electrical power delivered by the generator to the infinite bus during the fault

$$p_{e2} = \frac{E'V_{Th}}{X_{Th}}\sin\delta = \frac{(1.2812)(0.3333)}{0.46666}\sin\delta$$
$$= 0.9152\sin\delta \quad \text{per unit}$$

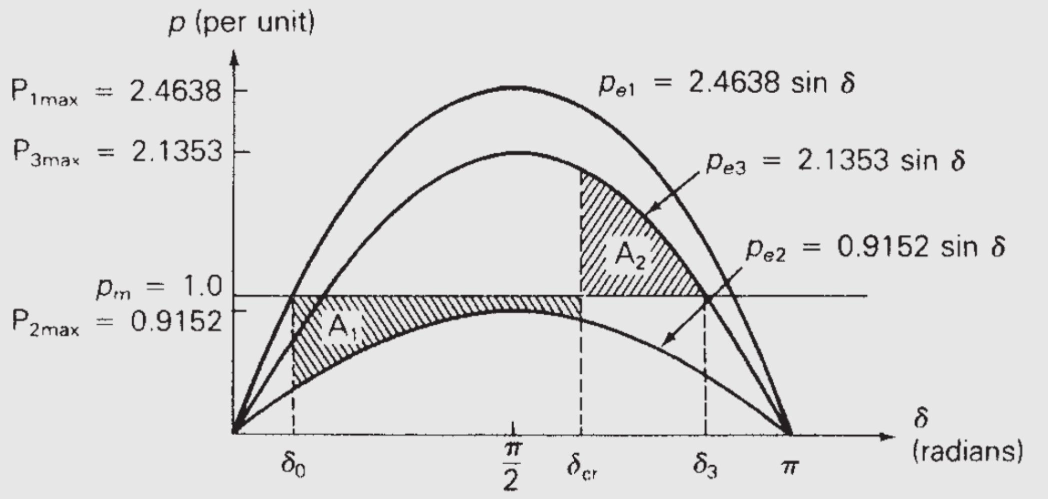
the postfault electrical power delivered, denoted

$$p_{e3} = \frac{(1.2812)(1.0)}{0.60}\sin\delta$$
$$= 2.1353\sin\delta \quad \text{per unit}$$

Example 3: electrical power curve changed

Solution:

The *p*–*δ* curves as well as the accelerating area A1 and decelerating area A2 corresponding to critical clearing are shown



$$A_1 = \int_{\delta_0}^{\delta_{cr}} (p_m - P_{2max} \sin\delta) d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{3max} \sin\delta - p_m) d\delta$$

$$\int_{0.4179}^{\delta_{cr}} (1.0 - 0.9152 \sin\delta) d\delta = \int_{\delta_{cr}}^{2.6542} (2.1353 \sin\delta - 1.0) d\delta$$

Solving for δ_{cr},

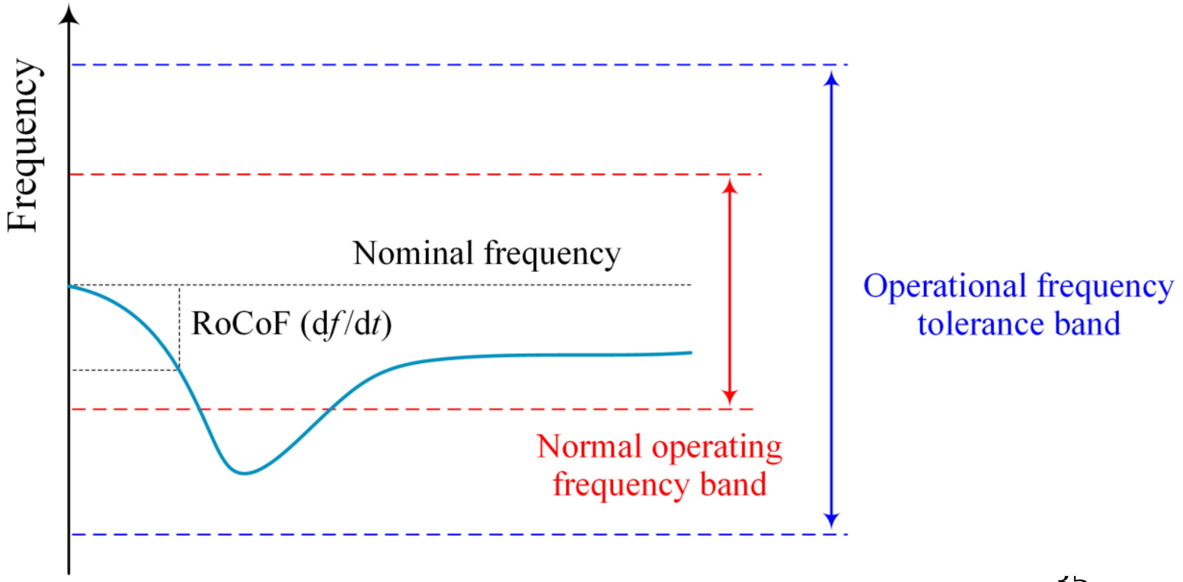
$$(\delta_{cr} - 0.4179) + 0.9152(\cos\delta_{cr} - \cos 0.4179)$$
$$= 2.1353(\cos\delta_{cr} - \cos 2.6542) - (2.6542 - \delta_{cr})$$
$$-1.2201 \cos\delta_{cr} = 0.4868$$
$$\delta_{cr} = 1.9812 \text{ radians} = 113.5^\circ$$

If the fault is cleared before $\delta = \delta_{cr} = 113.5°$, stability is maintained. Otherwise, stability is lost.

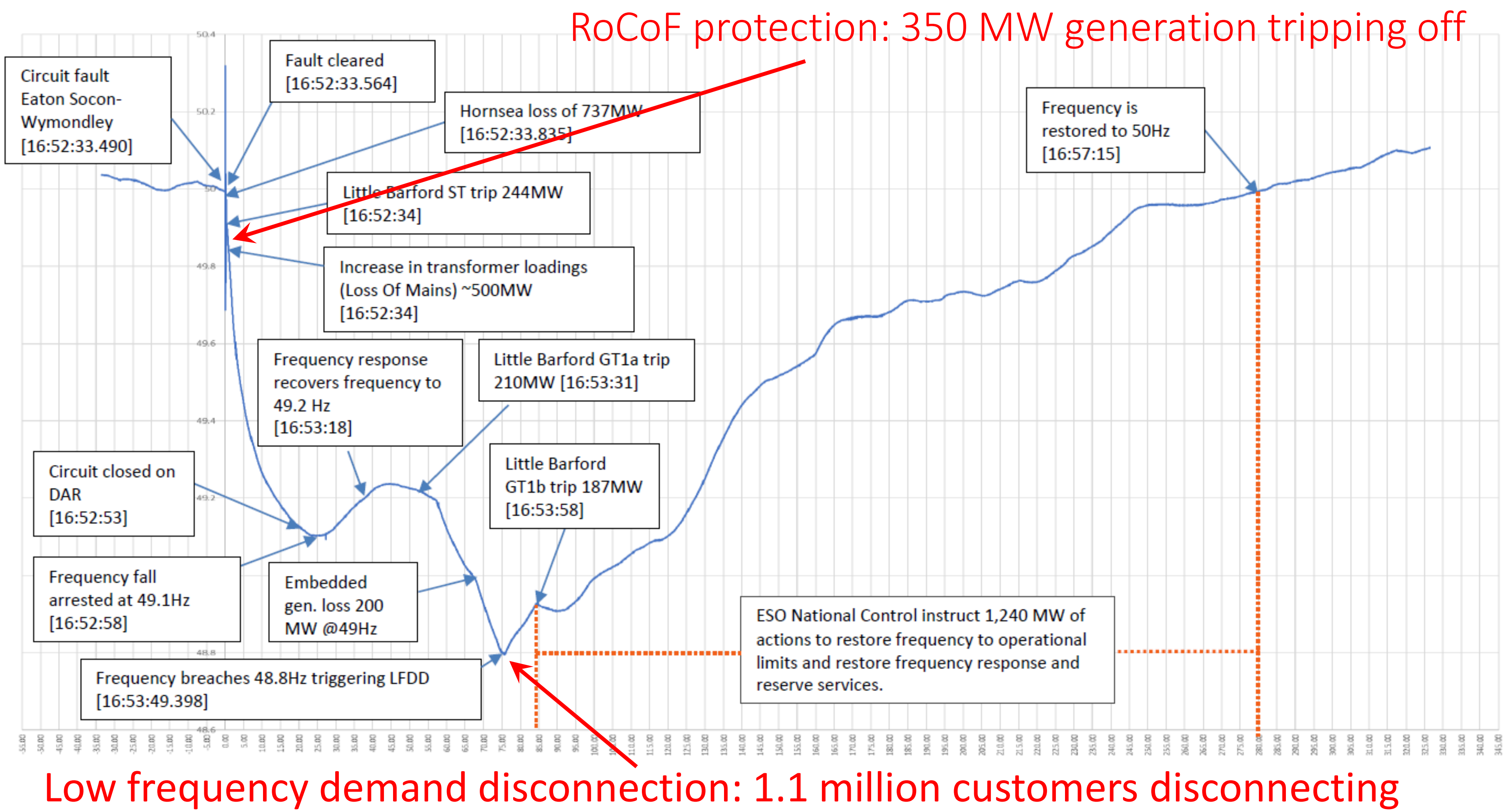
Frequency stability

	China	Australia
Nominal frequency	50 Hz	50 Hz
Normal operating frequency band	1) System ≥ 3 GW: ± 0.2 Hz 2) System < 3 GW: ± 0.5 Hz	1) Interconnected system: ± 0.15 Hz 2) Islanded system: ± 0.5 Hz
Operational frequency tolerance band	49-51 Hz	49-51 Hz (Extreme: 47-52 Hz)
RoCoF (df/dt) requirement		± 4 Hz/s for 0.25 s & ± 3 Hz/s for 1 s

Rate of Change of Frequency



British Blackout (August 9, 2019)



https://www.youtube.com/watch?v=eynO_SMgcag&ab_channel=Channel4News

Summary

- Synchronous generator operation
- Dynamical equations
- Three-phase fault
- Stability analysis
- Equal Area Criterion
- Applications