EE3123 Introduction to Electric Power Systems

Power Flow Analysis

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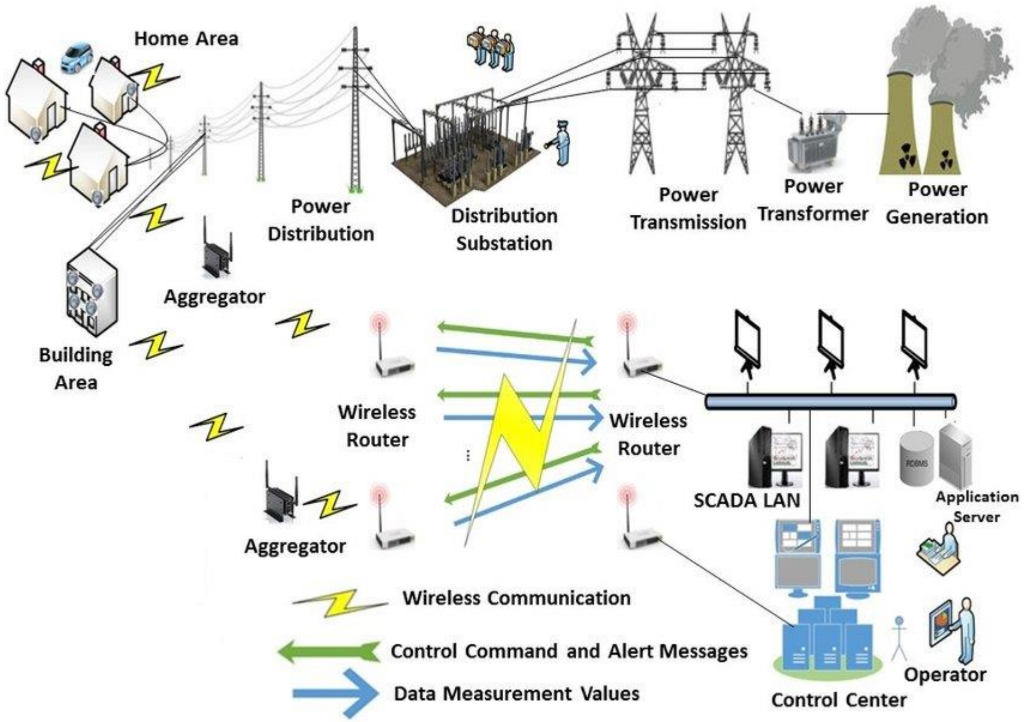
Many thanks to Prof. Michael Tse

What is power flow



Successful power system operation under normal balanced three-phase steady-state conditions requires the following:

1. Generation supplies the demand (load) plus losses.

2. Bus voltage magnitudes remain close to rated values.

3. Generators operate within specified real and reactive power limits.

4. Transmission lines and transformers are not overloaded.

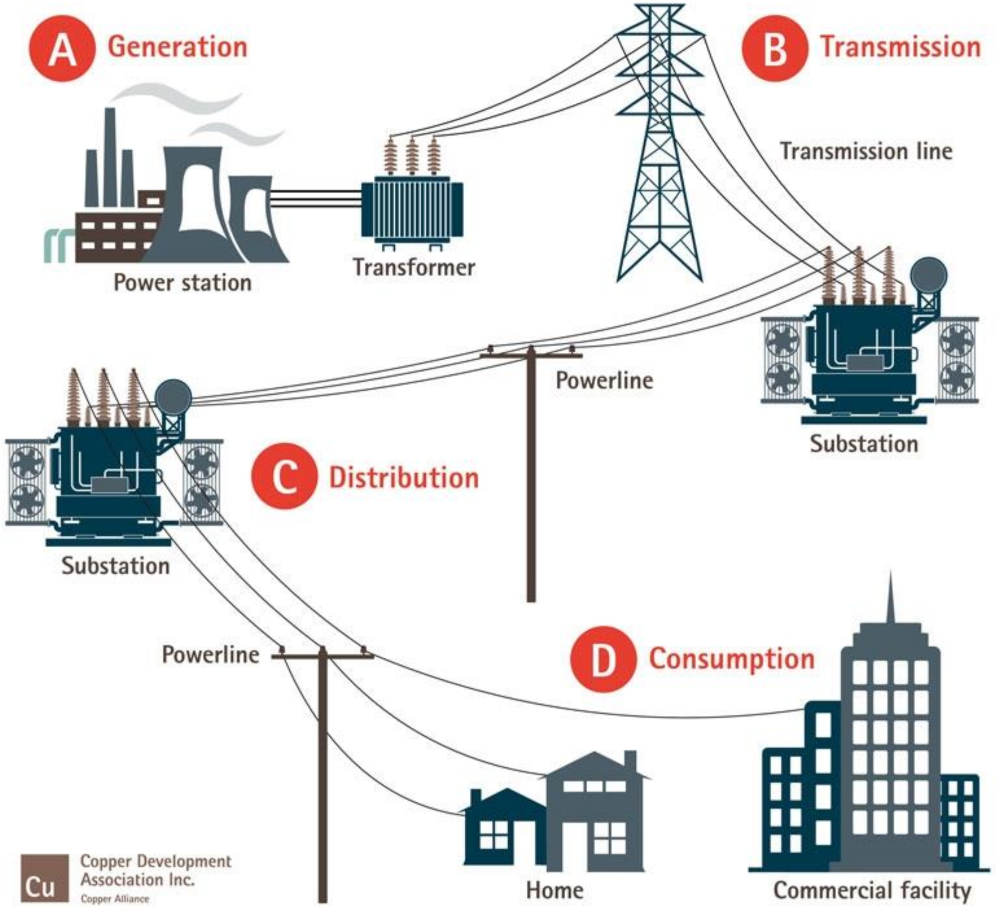
The power flow (sometimes also called the *load flow*) is the basic tool for investigating these requirements. The power flow determines the voltage magnitude and angle at each bus in a power system under balanced three-phase steady-state conditions.

It also computes real and reactive power flows for all equipment interconnecting the buses, as well as equipment losses.

Conventional nodal or loop analysis is not suitable for power flow studies because the input data for loads are normally given in terms of power, not impedance.

Also, generators are considered to be power sources, not voltage or current sources. The power flow problem is therefore formulated as a set of nonlinear algebraic equations suitable for computer solution.

Aims of power flow analysis

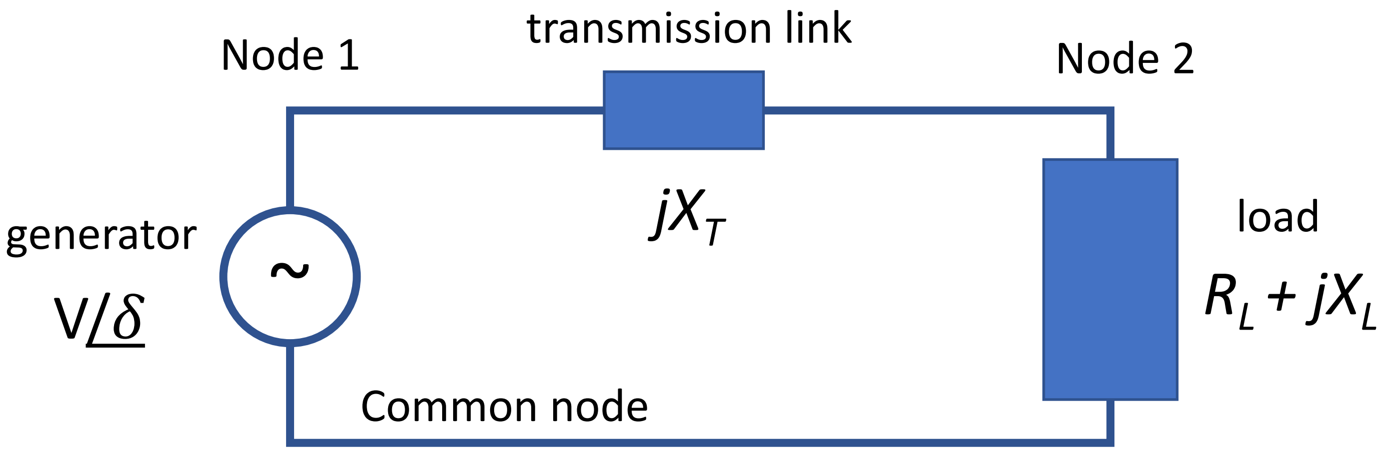


Power Flow Analysis is to find the power distribution and delivery in a network

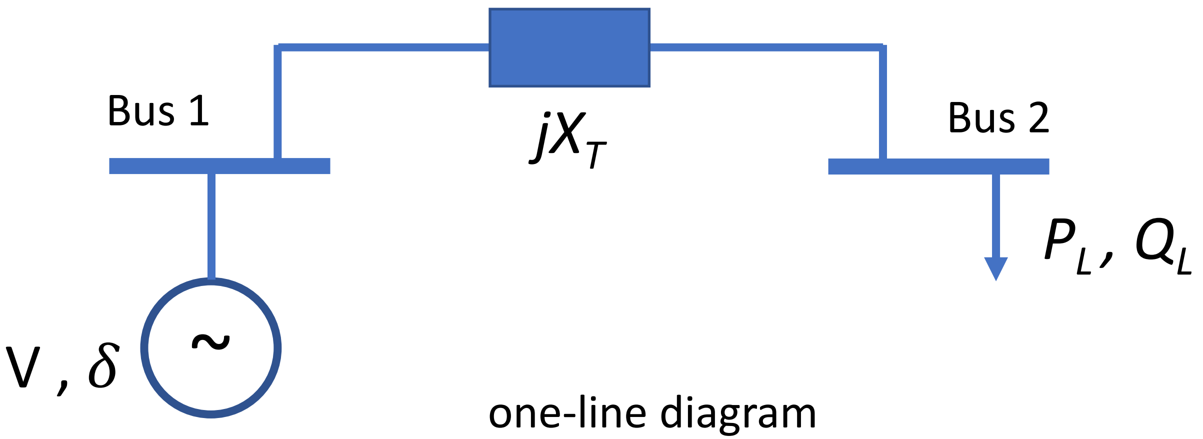
- To analyze load distribution
- To identify overloading points
- To facilitate planning of power generation
- To predict failure paths and fault impacts

Bus

- A “Bus” is defined as the connecting node of various components including a generator, a load and transmission line.
- A generator supplies power to the bus while a load absorbs power from the bus.
- A bus is considered as a node in a power network and thus the voltage is specified at each bus.

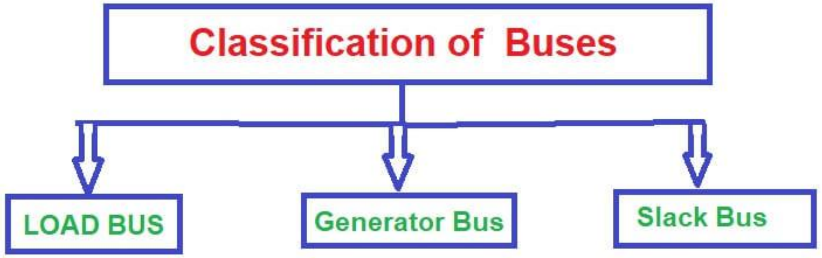


Electrical circuit
(complete with return path)



one-line diagram
(neglect return path and ground node)

Basics for power flow analysis



- Three types of bus in power systems:

  Load bus, generator bus and slack bus.

- <u>Load bus (PQ bus)</u> – Buses not having a generator
  - Real and reactive powers (P and Q) are specified
  - Bus voltage magnitude and phase angle (V and ϕ) will be calculated
  - Power supplied to the power system is positive
  - Power consumed from the system is negative.

- <u>Generator bus (PV bus)</u>
  - Voltage and real power supplied are specified
  - Bus phase angle will be calculated during iteration
  - Reactive power will be calculated after the case's solution is found

Basics for power flow analysis

- Slack bus (swing bus) – Special generator bus serving as the reference bus for the power system.
  - ***Voltage is fixed – both magnitude and phase.***
  - The bus supplies whatever real or reactive power is necessary to balance the power flow in the system.

Key points

- Voltage on a load bus (PQ bus) changes as the load varies – **P and Q are fixed**, while V (magnitude and angle) varies with load conditions.
- Generators (@ PV buses) operate normally with **P and V being maintained constant.**
- Slack bus generator varies P and Q to balance complex power – **V and angle reference are fixed**.

Power flow analysis

The analysis in normal steady-state operation is called a <u>**power flow study**</u> (load flow study) and it aims at determining the **Voltages** (magnitudes and phases), **Currents** and **Real** and **Reactive Power Flows** in a power system under specified generation and load conditions.

Minimal set of variables: V, δ, P, Q (other variables can be found from this set)

We know <u>**two variables**</u> of each bus (any 2 of **V**, δ, **P**, **Q**).

We find the <u>**other unknown variables**</u>.

Assumptions

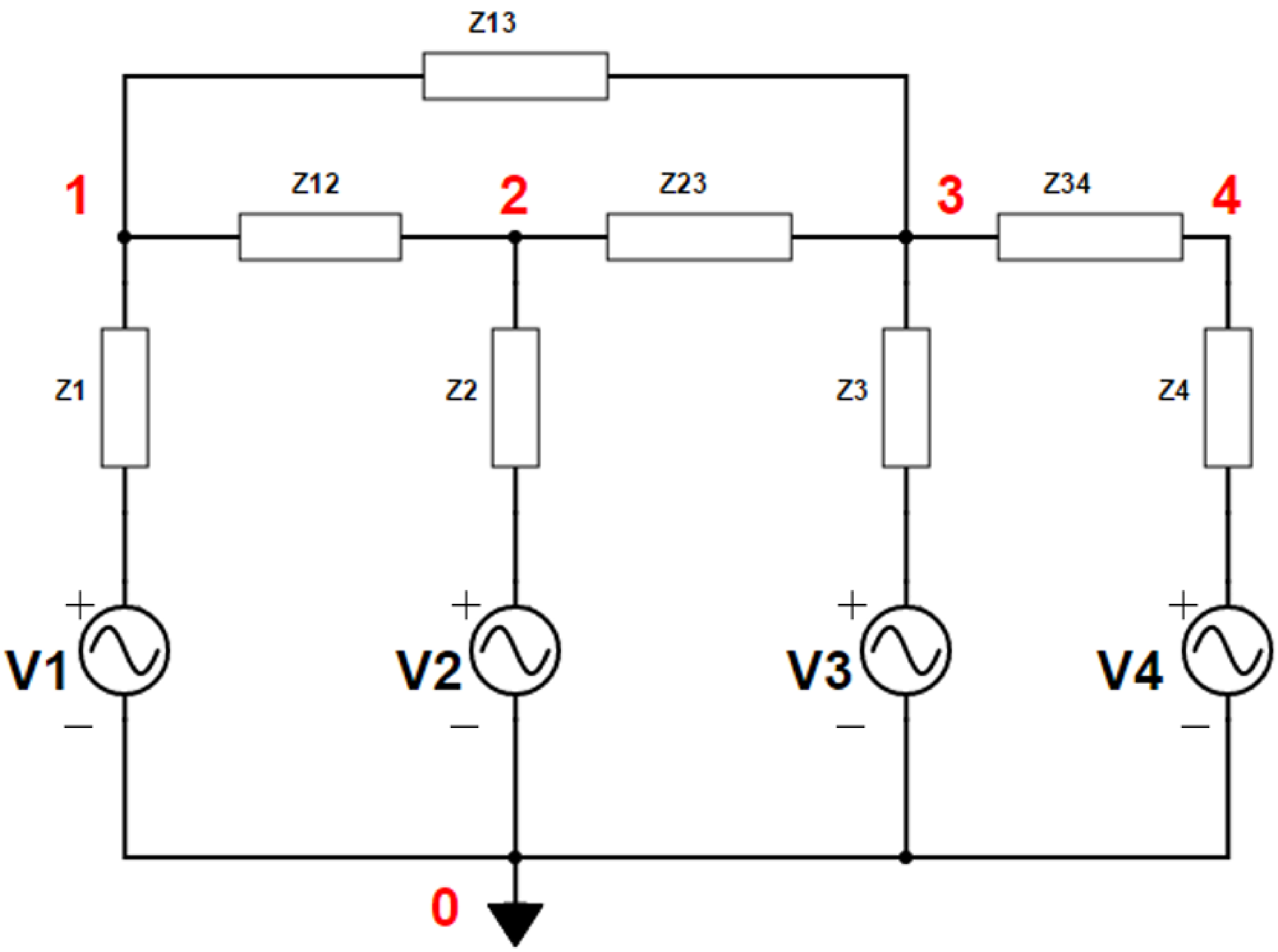
- Generation supply = load demand + system losses.
- Voltage magnitudes of buses remain close to rated values.
- Generators operate within specified real and reactive power limits.
- Transformers and transmission lines are not overloaded.

Analysis

- To represent the power system by a one-line diagram
- To determine the impedance in terms of information in the one-line diagram
- To formulate network equations and power flow equation
- To solve these equations.

Admittance matrix: 4-node example

Electrical circuit with 4 nodes and one reference node.



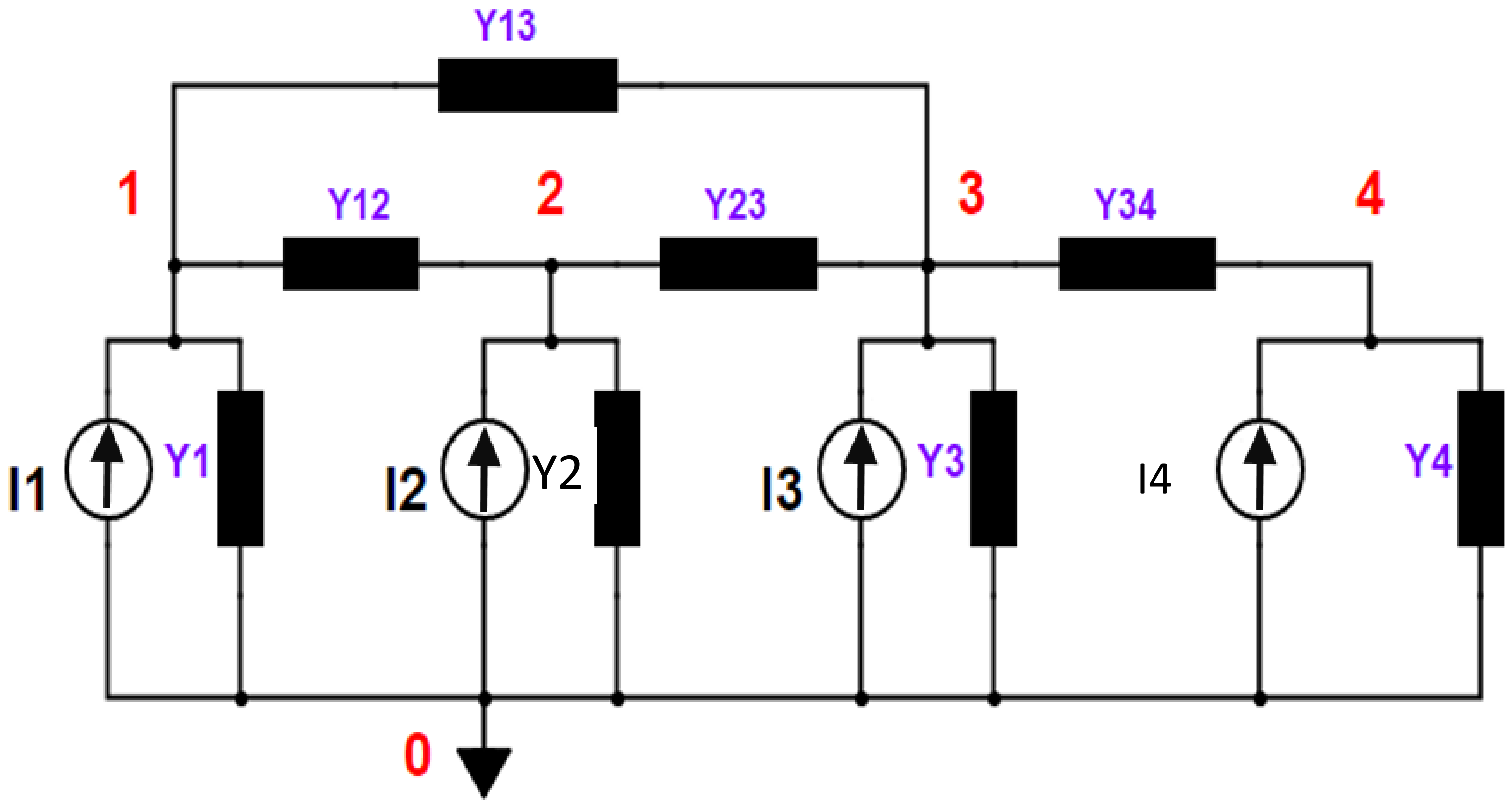
Admittance is defined as $Y \equiv \dfrac{1}{Z}$

Z is the impedance, measured in ohms

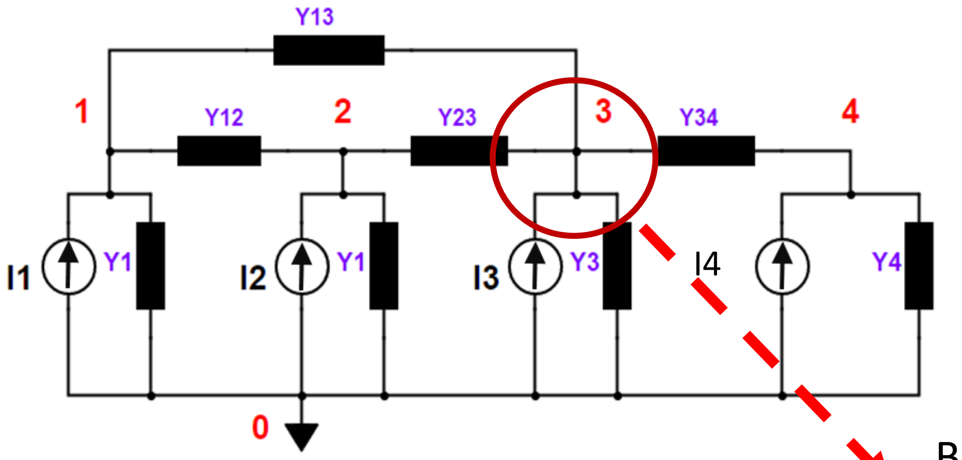
Admittance matrix: 4-node example

To perform nodal analysis, we convert to current source driving.

But we can also interpret the current source as current injected into the node. So, $I_n > 0$ if power is supplied to node n.



Admittance matrix: 4-node example

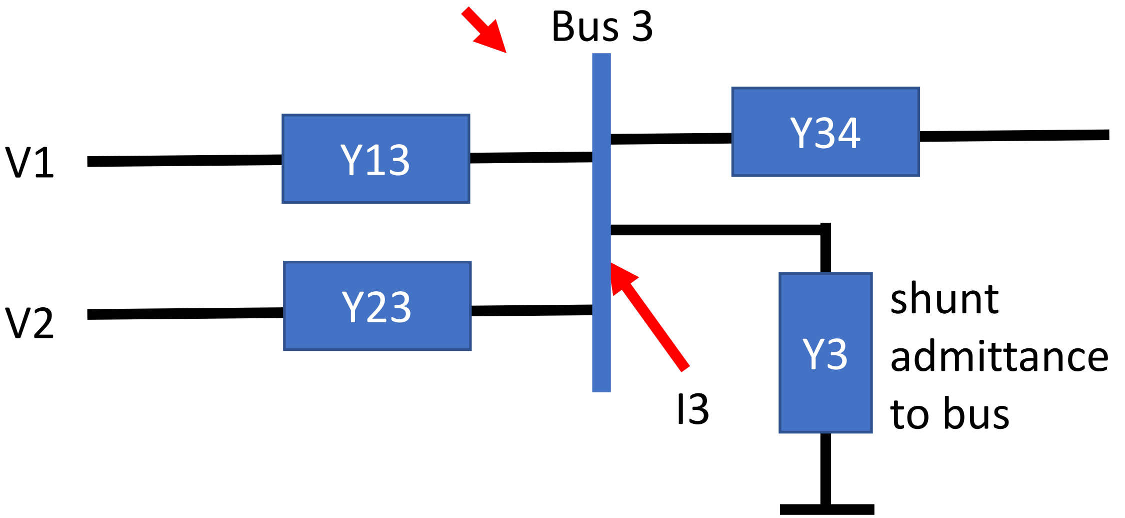


Consider bus 3:

$$I_3 = V_3 Y_3 + (V_3 - V_1) Y_{31} + (V_3 - V_2) Y_{32} + (V_3 - V_4) Y_{34}$$

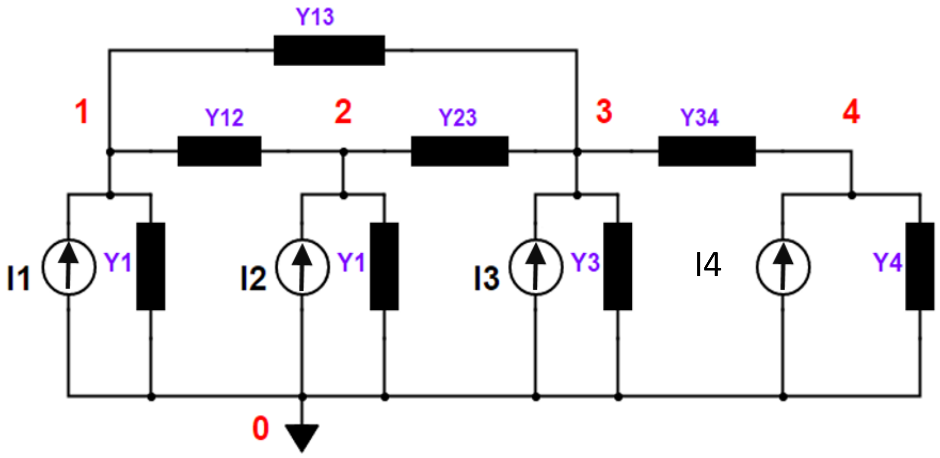
$$I_3 = V_3 \underbrace{(Y_3 + \sum_{m \neq 3} Y_{3m})}_{Y_{33}} - \sum_{m \neq 3} V_m Y_{3m}$$

Y_{33} = sum of all admittances connected to bus 3



$$I_3 = V_3 Y_{33} - \sum_{m \neq 3} V_m Y_{3m}$$

Admittance matrix: 4-node example



Repeat for all buses (just the usual **NODAL ANALYSIS**):

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} & 0 \\ -Y_{21} & Y_{22} & -Y_{23} & 0 \\ -Y_{31} & -Y_{32} & Y_{33} & -Y_{34} \\ 0 & 0 & -Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

NOTE: $Y_{mk} = Y_{km}$

admittance connecting nodes *m* and *k*

$$Y_{11} = Y_1 + Y_{12} + Y_{13}$$

$$Y_{22} = Y_2 + Y_{21} + Y_{23}$$

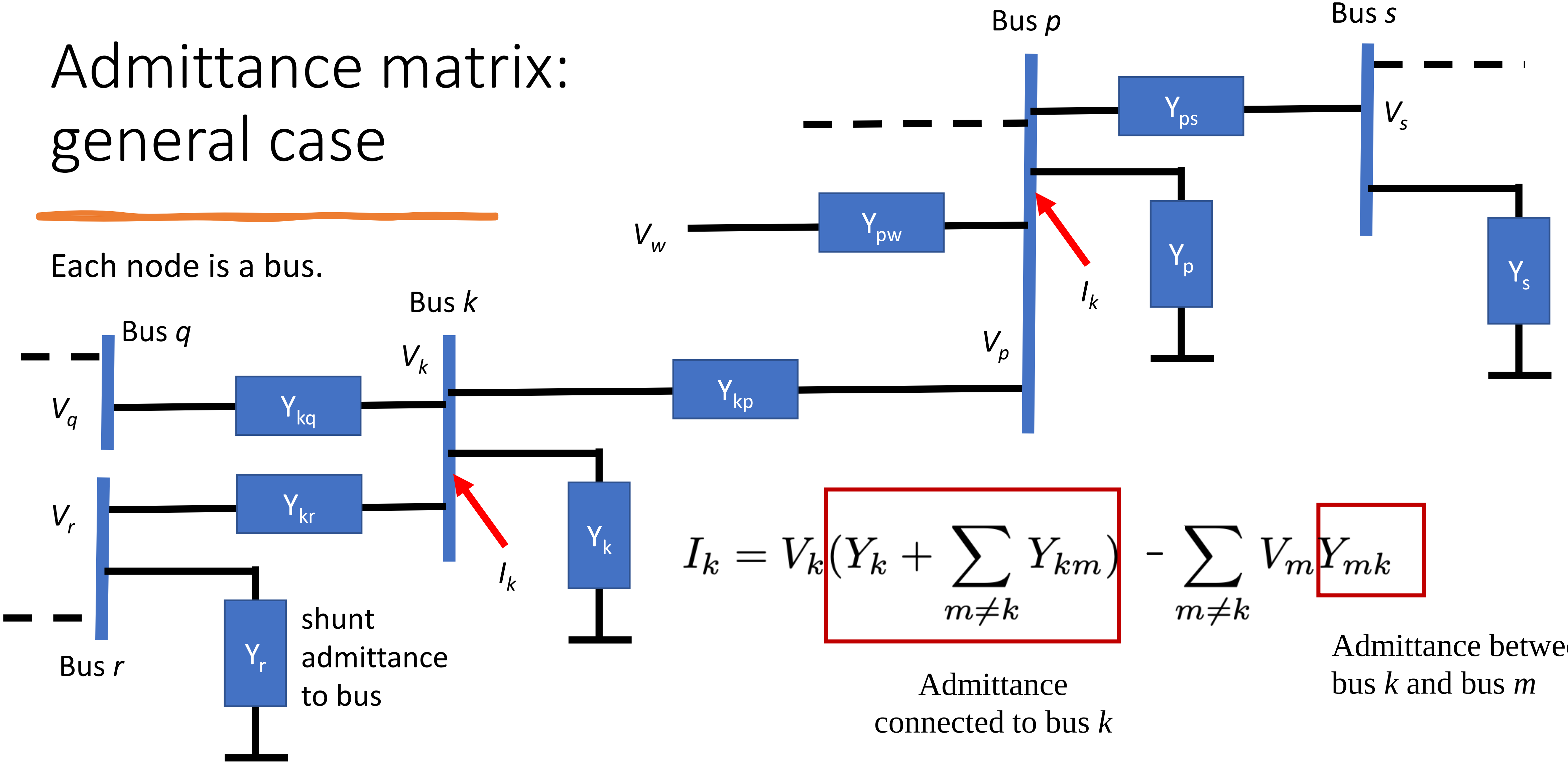
$$Y_{33} = Y_3 + Y_{31} + Y_{32} + Y_{34}$$

$$Y_{44} = Y_4 + Y_{43}$$

$$\text{OR} \;\; Y_{kk} = Y_k + \sum_{m \neq k} Y_{mk}$$

Admittance matrix: general case

Each node is a bus.



$$I_k = V_k \boxed{\left(Y_k + \sum_{m\neq k} Y_{km}\right)} - \sum_{m\neq k} V_m \boxed{Y_{mk}}$$

Admittance connected to bus k

Admittance between bus k and bus m

Admittance matrix: general case

$$\begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & -Y_{1n} \\ \vdots & \ddots & \vdots \\ -Y_{n1} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_n \end{bmatrix}$$

Y_{kk} = sum of all admittances connecting to node k

$Y_{kn} = Y_{nk}$ = sum of all admittances connecting node k and node n

Admittance matrix: Y_{bus}

The system matrix can be formed

$$I = Y_{\text{bus}} V$$

where Y_{bus} is the bus admittance matrix of order *n* x *n*;
V is the bus voltage vector;
I is the source current vector.

Bus voltages can also be solved as

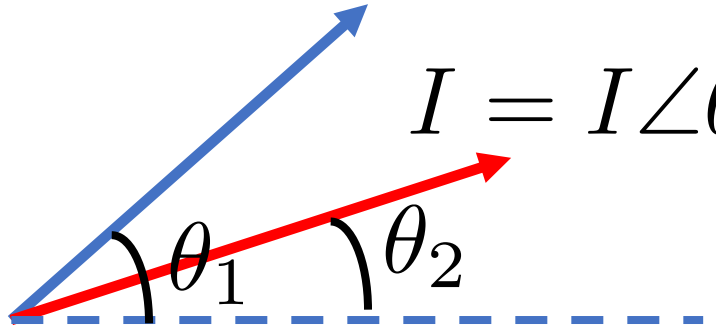
$$V = Y_{\text{bus}}^{-1} I$$

The bus voltages and currents can be used to calculate power flow.

Complex power

$$V = V\angle\theta_1$$

$$I = I\angle\theta_2$$



$$P = |V| \cdot |I| \cos(\theta_1 - \theta_2)$$
$$Q = |V| \cdot |I| \sin(\theta_1 - \theta_2)$$

Complex power is

$$S = P + jQ$$
$$S = |V|.|I|\angle(\theta_1 - \theta_2) = VI^*$$

$$S = VI^*$$

Power flow equations

The complex power at bus *k:*

$$S_k = V_k I_k^* = P_k + jQ_k$$

$$S_k = V_k \left[\sum_{m=1}^{n} Y_{mk} V_m \right]^* = V_k \sum_{m=1}^{n} Y_{mk}^* V_m^*$$

With the complex admittance, Y_{mk} is represented by

$$Y_{mk} = G_{mk} + jB_{mk}$$

where G_{mk} and B_{mk} are the real and imaginary parts of the admittance matrix element Y_{mk}

Power flow equations

We can rewrite the complex power as

$$S_k = V_k \angle \theta_k \sum_{m=1}^{n} (G_{mk} + jB_{mk})^* (V_m \angle \theta_m)^*$$

$$= \sum_{m=1}^{n} V_k V_m \angle (\theta_k - \theta_m)(G_{mk} - jB_{mk})$$

Using Euler relation, we have

$$S_k = \sum_{m=1}^{n} V_k V_m (\cos(\theta_k - \theta_m) + j \sin(\theta_k - \theta_m))(G_{mk} - jB_{mk})$$

$$P_k = \sum_{m=1}^{n} V_k V_m (G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m))$$

$$Q_k = \sum_{m=1}^{n} V_k V_m (G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m))$$

Solving the power flow equations

Suppose we have 100 buses.
We have 200 equations.

For each bus, if we know 2 variables, we have only 2 unknowns.

We have 200 equations, with 200 unknowns to solve.

$$P_k = \sum_{m=1}^{n} V_k V_m (G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m))$$

$$Q_k = \sum_{m=1}^{n} V_k V_m (G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m))$$

Solving the power flow equations

We have 2n equations to solve for 2n unknowns.

- unknowns : V_1, θ_1, P_2, Q_2, etc.

The equations are nonlinear, containing products and sine/cosine of unknowns!
Need numerical methods:

- Gauss-Seidel method
- Newton-Raphson method

$$P_k = \sum_{m=1}^{n} V_k V_m (G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m))$$

$$Q_k = \sum_{m=1}^{n} V_k V_m (G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m))$$

What is Gauss-Seidel method

- Consider the following set of linear algebraic equations in matrix format:

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & & \vdots & \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \Rightarrow \boxed{\mathbf{Ax} = \mathbf{y}}$$

where **x** and **y** are N vectors and **A** is an $N \times N$ square matrix. The components of **x**, **y**, and **A** may be real or complex. Given **A** and **y**, the objective is to solve for **x**.

- Iterative solutions: Gauss-Seidel

A general iterative solution to up equation proceeds as follows.
First select an initial guess **x**(0). Then use

$$\mathbf{x}(i+1) = \mathbf{g}[\mathbf{x}(i)] \qquad i = 0, 1, 2, \ldots$$

where **X**(i) is the ith guess and **g** is an N vector of functions that specify the iteration method.
Continue this procedure until the following stopping condition is satisfied, as

$$\left| \frac{x_k(i+1) - x_k(i)}{x_k(i)} \right| < \varepsilon \qquad \text{for all } k = 1, 2, \ldots, N$$

where $x_k(i)$ is the kth component of **X**(i) and ε is a specified *tolerance level.*

What is Gauss-Seidel method

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & & \vdots & \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}$$

$$y_k = A_{k1}x_1 + A_{k2}x_2 + \cdots + A_{kk}x_k + \cdots + A_{kN}x_N$$

$$\boxed{\begin{aligned} x_k &= \frac{1}{A_{kk}}[y_k - (A_{k1}x_1 + \cdots + A_{k,k-1}x_{k-1} + A_{k,k+1}x_{k+1} + \cdots + A_{kN}x_N)] \\ &= \frac{1}{A_{kk}}\left[y_k - \sum_{n=1}^{k-1} A_{kn}x_n - \sum_{n=k+1}^{N} A_{kn}x_n\right] \end{aligned}}$$

where **x** and **y** are *N* vectors and **A** is an $N \times N$ square matrix. The components of **x**, **y**, and **A** may be real or complex. Given **A** and **y**, the objective is to solve for **x**.

- The Gauss-Seidel method is given by

$$x_k(i+1) = \frac{1}{A_{kk}}\left[y_k - \sum_{n=1}^{k-1} A_{kn}x_n(i+1) - \sum_{n=k+1}^{N} A_{kn}x_n(i)\right]$$

- Give an initial value to **x**, and then use Gauss-Seidel to calculate **x**(*i+1*) by iterations until fulfill

$$\left|\frac{x_k(i+1) - x_k(i)}{x_k(i)}\right| < \varepsilon \qquad \text{for all } k = 1, 2, \ldots, N$$

What is Gauss-Seidel method

- ❑ Example

Solve

$$\begin{bmatrix} 10 & 5 \\ 2 & 9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

- ❑ Gauss-Seidel method: iterative solution to linear algebraic equations

$$x_k(i+1) = \frac{1}{A_{kk}}\left[y_k - \sum_{n=1}^{k-1} A_{kn}x_n(i+1) - \sum_{n=k+1}^{N} A_{kn}x_n(i)\right]$$

$$k=1 \quad x_1(i+1) = \frac{1}{A_{11}}[y_1 - A_{12}x_2(i)] = \frac{1}{10}[6 - 5x_2(i)]$$

$$k=2 \quad x_2(i+1) = \frac{1}{A_{22}}[y_2 - A_{21}x_1(i+1)] = \frac{1}{9}[3 - 2x_1(i+1)]$$

Using this equation for $x_1(i+1)$, $x_2(i+1)$ also can be written as

$$x_2(i+1) = \frac{1}{9}\left\{3 - \frac{2}{10}[6 - 5x_2(i)]\right\}$$

Starting with $x_1(0) = x_2(0) = 0$, the solution is given in the following table:

Gauss-Seidel

i	0	1	2	3	4	5	6
$x_1(i)$	0	0.60000	0.50000	0.48889	0.48765	0.48752	0.48750
$x_2(i)$	0	0.20000	0.22222	0.22469	0.22497	0.22500	0.22500

What is Newton-Raphson method

- A set of nonlinear algebraic equations in matrix format is given by

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_N(\mathbf{x}) \end{bmatrix} = \mathbf{y} \Rightarrow \boxed{0 = \mathbf{y} - \mathbf{f}(\mathbf{x})}$$

where **y** and **x** are *N* vectors and **f**(**x**) is an *N* vector of functions. Given **y** and **f**(**x**), the objective is to solve for **x**.

Adding **Dx** to both sides, where **D** is a square *N* x *N* invertible matrix.

$$\mathbf{D}\mathbf{x} = \mathbf{D}\mathbf{x} + \mathbf{y} - \mathbf{f}(\mathbf{x}) \Rightarrow \boxed{\mathbf{x} = \mathbf{x} + \mathbf{D}^{-1}[\mathbf{y} - \mathbf{f}(\mathbf{x})]} \Rightarrow \mathbf{x}(i+1) = \mathbf{x}(i) + \mathbf{D}^{-1}\{\mathbf{y} - \mathbf{f}[\mathbf{x}(i)]\}$$

One method for specifying **D**, called ***Newton-Raphson***, is based on the following Taylor series expansion of **f**(**x**) about an operating point $\mathbf{x}_0$.

$$\mathbf{y} = \mathbf{f}(\mathbf{x}_0) + \left.\frac{d\mathbf{f}}{d\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_0}(\mathbf{x} - \mathbf{x}_0)\cdots \Rightarrow \mathbf{x} = \mathbf{x}_0 + \left[\left.\frac{d\mathbf{f}}{d\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_0}\right]^{-1}[\mathbf{y} - \mathbf{f}(\mathbf{x}_0)]$$

Neglecting higher order terms

What is Newton-Raphson method

- The Newton-Raphson method replaces $\mathbf{x}_0$ by the old value $\mathbf{x}(i)$ and $\mathbf{x}$ by the new value $\mathbf{x}(i+1)$

$$\boxed{\mathbf{x}(i+1) = \mathbf{x}(i) + \mathbf{J}^{-1}(i)\{\mathbf{y} - \mathbf{f}[\mathbf{x}(i)]\}}$$

$$\mathbf{J}(i) = \left.\frac{d\mathbf{f}}{d\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}(i)} = \left[\begin{array}{cccc} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_N} \end{array}\right]_{\mathbf{x}=\mathbf{x}(i)}$$

The *N x N* matrix $\mathbf{J}(i)$, whose elements are the partial derivatives, is called the Jacobian matrix.

What is Newton-Raphson method

- Example: Newton-Raphson method - solution to nonlinear algebraic equations

Solve

$$\begin{bmatrix} x_1 + x_2 \\ x_1 x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 50 \end{bmatrix} \qquad \mathbf{x}(0) = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

Use the Newton-Raphson method starting with the above **x**(0) and continue until it is satisfied with $\varepsilon = 10^{-4}$.

Using Jacobian matrix with $f_1 = (x_1 + x_2)$ and $f_2 = x_1 x_2$

$$\mathbf{J}(i)^{-1} = \left[\begin{array}{c|c} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \hline \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{array}\right]^{-1}_{\mathbf{x}=\mathbf{x}(i)} = \left[\begin{array}{c|c} 1 & 1 \\ \hline x_2(i) & x_1(i) \end{array}\right]^{-1} = \frac{\left[\begin{array}{c|c} x_1(i) & -1 \\ \hline -x_2(i) & 1 \end{array}\right]}{x_1(i) - x_2(i)}$$

$$\boxed{\mathbf{x}(i+1) = \mathbf{x}(i) + \mathbf{J}^{-1}(i)\{\mathbf{y} - \mathbf{f}[\mathbf{x}(i)]\}}$$

$$\begin{bmatrix} x_1(i+1) \\ x_2(i+1) \end{bmatrix} = \begin{bmatrix} x_1(i) \\ x_2(i) \end{bmatrix} + \frac{\left[\begin{array}{c|c} x_1(i) & -1 \\ \hline -x_2(i) & 1 \end{array}\right]}{x_1(i) - x_2(i)} \begin{bmatrix} 15 - x_1(i) - x_2(i) \\ 50 - x_1(i)x_2(i) \end{bmatrix}$$

What is Newton-Raphson method

- Example: Newton-Raphson method - solution to nonlinear algebraic equations

Solve

$$\begin{bmatrix} x_1 + x_2 \\ x_1 x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 50 \end{bmatrix} \qquad \mathbf{x}(0) = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

Use the Newton-Raphson method starting with the above **x**(0) and continue until it is satisfied with $\varepsilon=10^{-4}$.

Writing the preceding as two separate equations,

$$x_1(i+1) = x_1(i) + \frac{x_1(i)[15 - x_1(i) - x_2(i)] - [50 - x_1(i)x_2(i)]}{x_1(i) - x_2(i)}$$

$$x_2(i+1) = x_2(i) + \frac{-x_2(i)[15 - x_1(i) - x_2(i)] + [50 - x_1(i)x_2(i)]}{x_1(i) - x_2(i)}$$

Successive calculations of these equations are shown in the following table.

Newton-Raphson

i	0	1	2	3	4
$x_1(i)$	4	5.20000	4.99130	4.99998	5.00000
$x_2(i)$	9	9.80000	10.00870	10.00002	10.00000

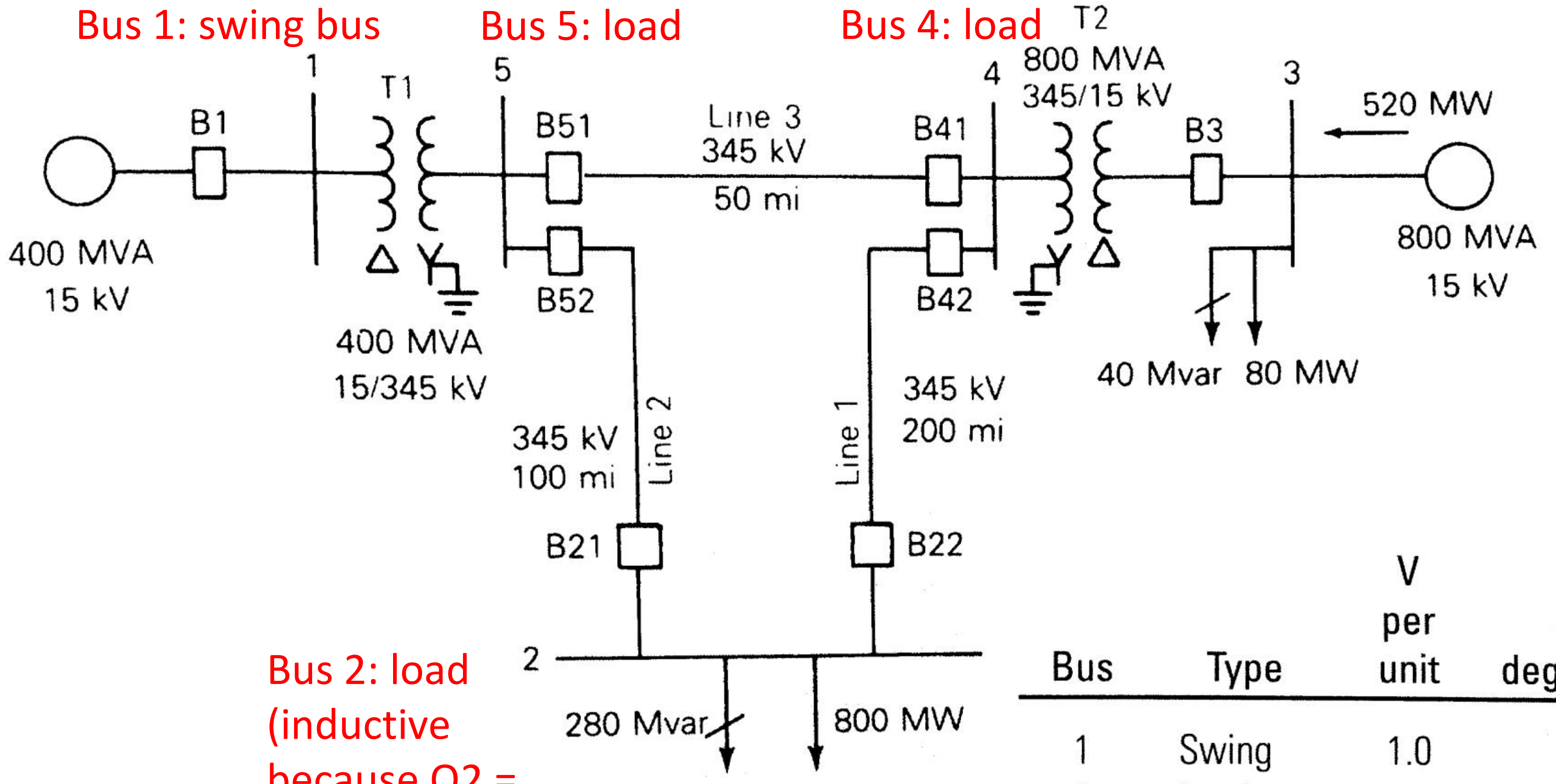
Newton-Raphson converges in four iterations for this example.

Practical Example

Bus 1: swing bus

Bus 5: load

Bus 4: load



Bus 3: generator
and load (inductive as QL3 = –0.4)

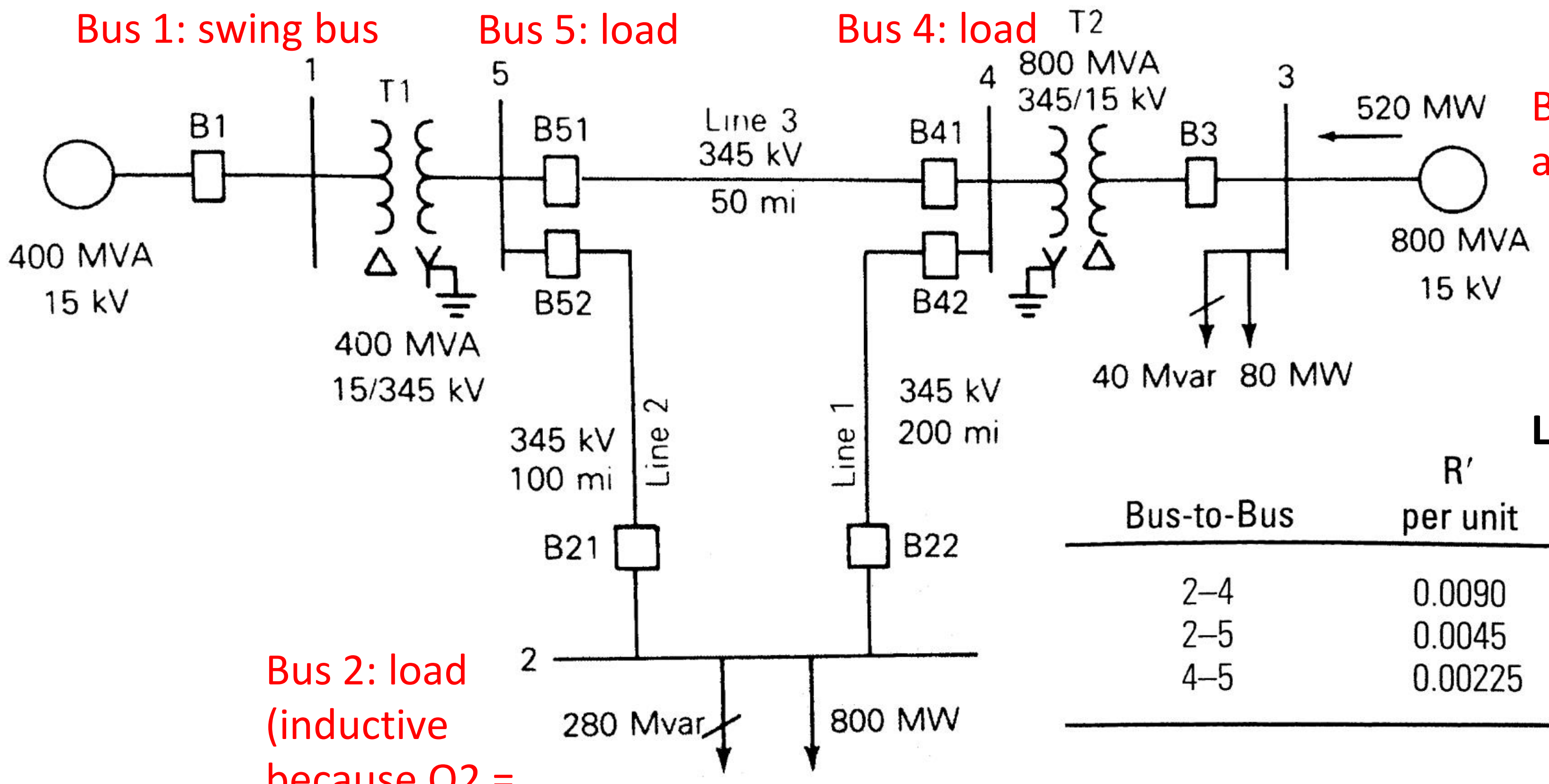
Bus 2: load
(inductive
because Q2 =
–2.8 pu)

S_{base} = 100 MVA, V_{base} = 15 kV at buses 1, 3
and V_{base} = 345 kV at buses 2, 4, 5

BUS INPUT DATA

Bus	Type	V per unit	δ degrees	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	−2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

Solving the power flow equations



Bus 3: generator
and load (inductive as QL3 = –0.4)

Bus 2: load (inductive because Q2 = –2.8 pu)

Shunt cap to bus for pf enhancement

LINE DATA

Bus-to-Bus	R′ per unit	X′ per unit	G′ per unit	B′ per unit	Maximum MVA per unit
2–4	0.0090	0.100	0	1.72	12.0
2–5	0.0045	0.050	0	0.88	12.0
4–5	0.00225	0.025	0	0.44	12.0

TRANSFORMER DATA

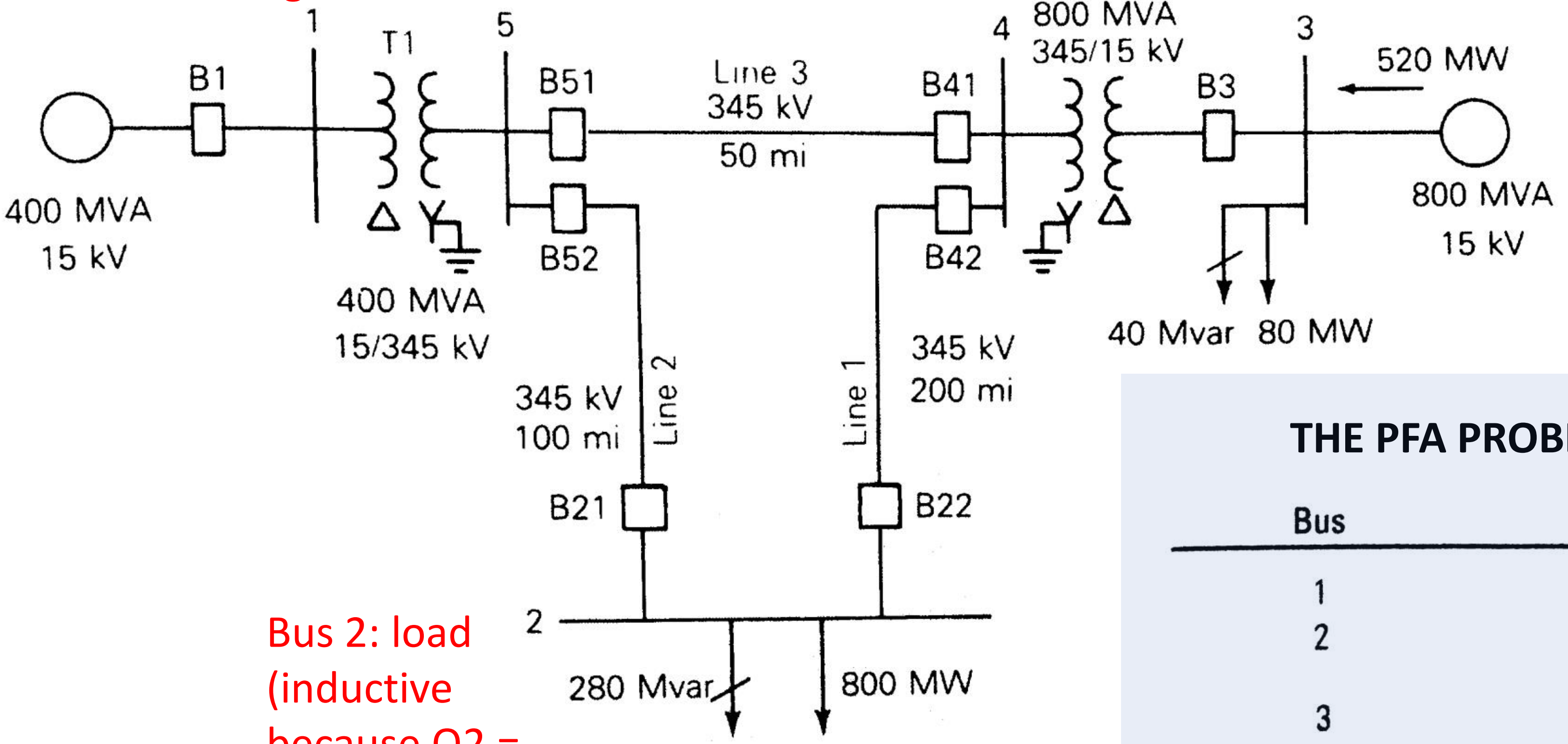
Bus-to-Bus	R per unit	X per unit	G_c per unit	B_m per unit	Maximum MVA per unit	Maximum TAP Setting per unit
1–5	0.00150	0.02	0	0	6.0	—
3–4	0.00075	0.01	0	0	10.0	—

Practical Example

Bus 1: swing bus

Bus 5: load

Bus 4: load



Bus 3: generator and load (inductive as QL3 = –0.4)

Bus 2: load (inductive because Q2 = –2.8 pu)

THE PFA PROBLEM: UNKNOWNS TO BE FOUND

Bus	Input Data	Unknowns
1	$V_1 = 1.0, \delta_1 = 0$	P_1, Q_1
2	$P_2 = P_{G2} - P_{L2} = -8$ $Q_2 = Q_{G2} - Q_{L2} = -2.8$	V_2, δ_2
3	$V_3 = 1.05$ $P_3 = P_{G3} - P_{L3} = 4.4$	Q_3, δ_4
4	$P_4 = 0, Q_4 = 0$	V_4, δ_4
5	$P_5 = 0, Q_5 = 0$	V_5, δ_5

Practical Example

$$Y_{\text{bus}}$$

(all in pu)

$$Y_{21} = Y_{23} = 0$$

$$Y_{24} = \frac{-1}{R_{24} + jX_{24}} = \frac{-1}{0.009 + j0.1} = -0.89276 + j9.91964$$

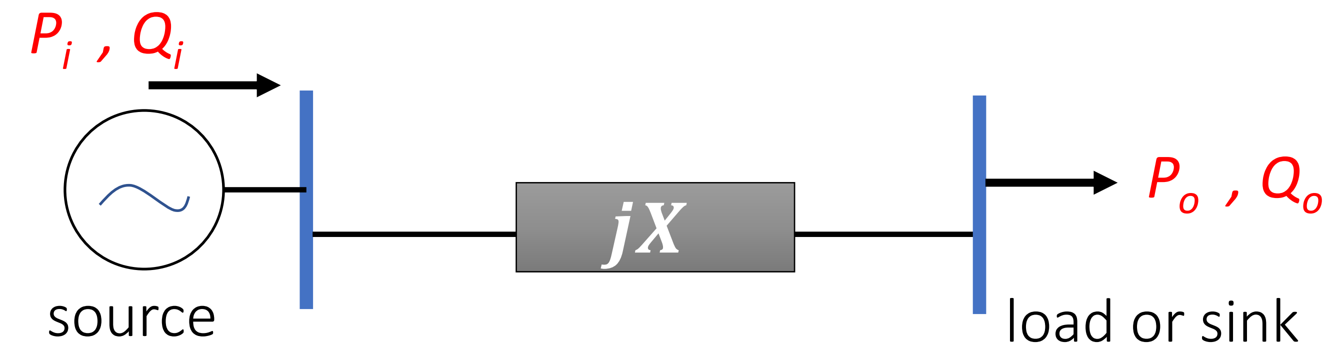
$$Y_{25} = \frac{-1}{R_{25} + jX_{25}} = \frac{-1}{0.0045 + j0.05} = -1.78552 + j19.83932$$

$$\begin{aligned} Y_{22} &= \frac{1}{R_{24} + jX_{24}} + \frac{1}{R_{25} + jX_{25}} + \frac{jB_{24}}{2} + \frac{jB_{25}}{2} \\ &= \frac{1}{R_{24} + jX_{24}} + \frac{1}{R_{25} + jX_{25}} + \frac{j1.72}{2} + \frac{j0.88}{2} \\ &= 2.67828 - j28.459 - 1.78552 + j19.83932 \end{aligned}$$

etc.

Then, computer program with numerical methods is used to solve for the unknowns.

Simple example: 2-bus system



$v_i = V_i \angle \delta$ $\qquad$ $v_o = V_o \angle 0$

Unknowns are P_i, Q_i, P_o, Q_o

We need to set up 4 equations.
First we find all current (only one here) using I = Y_{bus}V.
This is trivial for this example: $I = (V_i - V_o)/jX$.
Then, we have the power flow equations as

$$S_i = V_i I^*$$

$$S_o = V_o I^*$$

PFA solution

$$S_i = \frac{V_i V_o}{X} \sin\delta + j\left[\frac{V_i^2}{X} - \frac{V_i V_o}{X}\cos\delta\right]$$

$$S_o = \frac{V_i V_o}{X} \sin(\delta) + j\left[\frac{V_i V_o}{X}\cos(\delta) - \frac{V_o^2}{X}\right]$$

$$P_i = \frac{V_i V_o}{X}$$

$$Q_i = -\frac{V_i V_o}{X}\cos\delta + \frac{V_i^2}{X}$$

$$P_o = \frac{V_i V_o}{X}\sin\delta$$

$$Q_o = \frac{V_i V_o}{X}\cos\delta - \frac{V_o^2}{X}$$

That's easy! But if the unknowns are Q_i, δ, P_o, Q_o

4 equations, 4 unknowns

$$P_i = \frac{V_i V_o}{X} \sin \delta$$

$$Q_i = -\frac{V_i V_o}{X} \cos \delta + \frac{V_i^2}{X}$$

$$P_o = \frac{V_i V_o}{X} \sin \delta$$

$$Q_o = \frac{V_i V_o}{X} \cos \delta - \frac{V_o^2}{X}$$

We need to solve for the unknowns.

Approximate solution:

If δ is small, we may write $\sin \delta \approx \delta$, $\cos \delta \approx 1$.

$$P_i \approx \frac{V_i V_o}{X} \delta \quad \Rightarrow \quad \delta \approx \frac{P_i X}{V_i V_o}$$

$$Q_i \approx \frac{V_i (V_i - V_o)}{X}$$

$$P_o = P_i$$

$$Q_o \approx \frac{V_o (V_i - V_o)}{X}$$

SIMPLIFIED approach: DC power flow analysis

Assumption 1:

The resistance in all connecting lines is much less than the reactance on the transmission lines, i.e., $G_{km} \approx 0$, then we have

$$P_k = \sum_{m=1}^{n} V_k V_m (B_{km} \sin(\theta_k - \theta_m))$$

$$Q_k = \sum_{m=1}^{n} V_k V_m (-B_{km} \cos(\theta_k - \theta_m))$$

Assumption 2:

The angle difference between bus voltages is very small.

$$P_k = \sum_{m=1}^{n} V_k V_m (B_{km}(\theta_k - \theta_m))$$

$$Q_k = \sum_{m=1}^{n} V_k V_m (-B_{km})$$

$$Q_i = -b_k + \sum_{j=1, j\neq k}^{N} |b_{kj}|(|V_k| - |V_j|)$$

In Q_k, $V_k(V_k - V_m)$ is retained → $Q_k \approx 0$.

Assumption 3:

Bus voltage is nearly 1 pu, i.e., voltages are similar in magnitude.
Focus on real power because P >> Q.

$$P_k = \cancel{V_k^2(B_{km}(\underline{\theta_k - \theta_k})))}^{\,0} + \sum_{m=1, m\neq k}^{n} V_k V_m (B_{km}(\theta_k - \theta_m))$$

$$= \sum_{m=1, m\neq k}^{n} (B_{km}(\theta_k - \theta_m))$$

1pu

SIMPLIFIED approach: DC power flow analysis

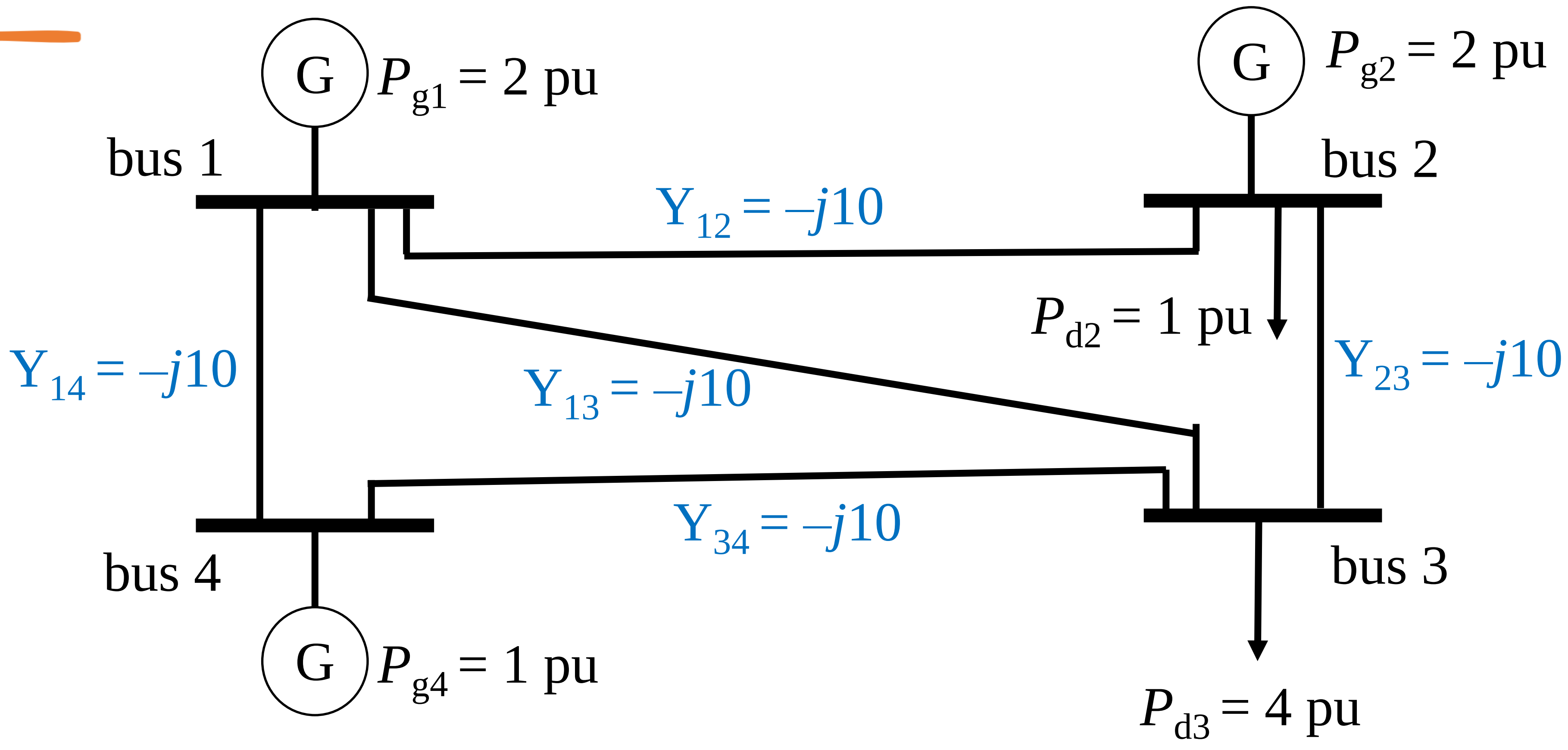
Linear equations.

Solvable without using numerical methods.

$$P_k = \sum_{m=1, m \neq k}^{n} \left(B_{km}(\theta_k - \theta_m)\right)$$

Because the equation is a linear equation with a form similar to that found in solving dc resistive circuits, this technique is referred to as the dc power flow. However, in contrast to the previous power flow algorithms, the dc power flow only gives an approximate solution with the degree of approximation system dependent. Nevertheless, with the advent of power system restructuring, the dc power flow has become a commonly used analysis technique.

Example



Solve this network using DC PFA without using numerical solution.

Assume the shunt admittance at all buses is zero, i.e., only line admittances are counted.

Example

$$P_k = \sum_{m=1, m \neq k}^{n} (B_{km}(\theta_k - \theta_m))$$

Write down the power equation for each bus:

$$\begin{aligned}
P_1 &= B_{12}(\theta_1 - \theta_2) + B_{13}(\theta_1 - \theta_3) + B_{14}(\theta_1 - \theta_4) \\
&= (B_{12} + B_{13} + B_{14})\theta_1 - B_{12}\theta_2 - B_{13}\theta_3 - B_{14}\theta_4 \\
P_2 &= -B_{21}\theta_1 + (B_{21} + B_{23} + B_{24})\theta_2 - B_{23}\theta_3 - B_{24}\theta_4 \\
P_3 &= -B_{31}\theta_1 - B_{32}\theta_2 + (B_{31} + B_{32} + B_{34})\theta_3 - B_{34}\theta_4 \\
P_4 &= -B_{41}\theta_1 - B_{42}\theta_2 - B_{43}\theta_3 + (B_{41} + B_{42} + B_{43})\theta_4
\end{aligned}$$

Solution

$$Y_{bus}$$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} B_{12}+B_{13}+B_{14} & -B_{12} & -B_{13} & -B_{14} \\ -B_{21} & B_{21}+B_{23}+B_{24} & -B_{23} & -B_{24} \\ -B_{31} & -B_{32} & B_{31}+B_{32}+B_{34} & -B_{34} \\ -B_{41} & -B_{42} & -B_{43} & B_{41}+B_{42}+B_{43} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 30 & -10 & -10 & -10 \\ -10 & 20 & -10 & 0 \\ -10 & -10 & 30 & -10 \\ -10 & 0 & -10 & 20 \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

Choose $\theta_1 = 0$ as reference to reduce order.

$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.025 \\ -0.15 \\ -0.025 \end{bmatrix}$$

Using $P_{km} = B_{km}(\theta_k - \theta_m)$
we get the power flow on each line as

$$P_{12} = B_{12}(\theta_1 - \theta_2) = 0.25 \text{ p.u.}$$
$$P_{13} = B_{13}(\theta_1 - \theta_3) = 1.5 \text{ p.u.}$$
$$P_{14} = B_{14}(\theta_1 - \theta_4) = 0.25 \text{ p.u.}$$
$$P_{23} = B_{23}(\theta_2 - \theta_3) = 1.25 \text{ p.u.}$$
$$P_{34} = B_{34}(\theta_3 - \theta_4) = -1.25 \text{ p.u.}$$

General Procedure of DC PFA

DC power flow equation : $$[P] = [B][\theta]$$

Power flows through branches are represented as

$$[P_{\mathrm{br}}] = ([b] \times [A])[\theta]$$

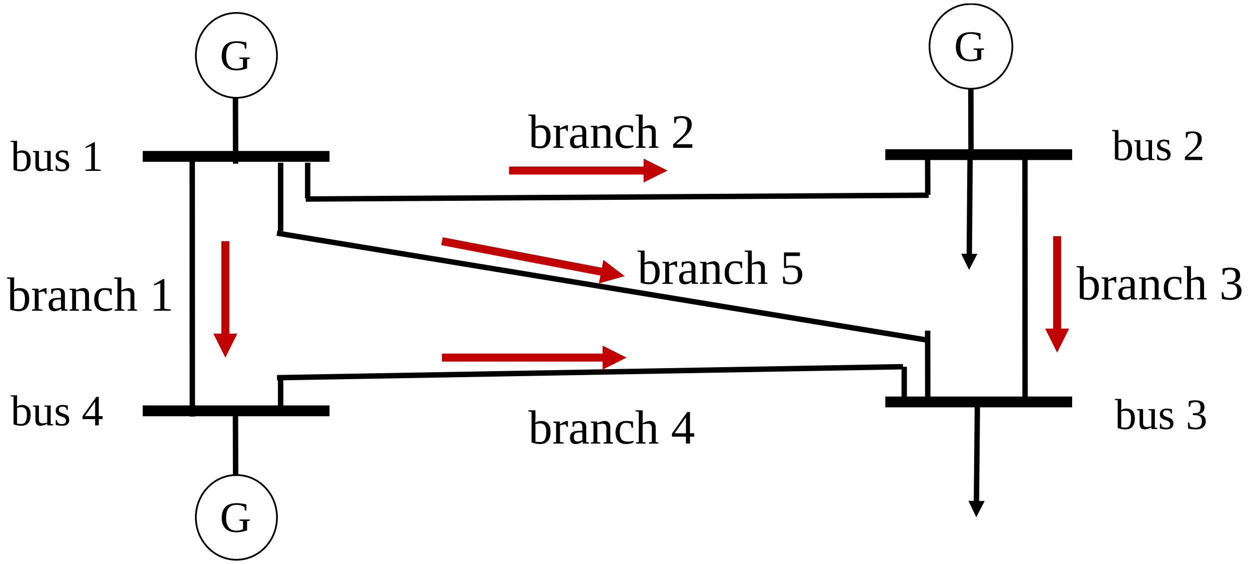
where:

- $[P]$: n-dim vector of bus active power injections for buses 1, ..., n
- $[B]$: n x n admittance matrix with R = 0
- $[\theta]$: n-dim vector of bus voltage angles for buses 1, ..., n
- $[P_{br}]$: n-dim vector of branch flows (M is the number of branches)
- $[b]$: M x M matrix (b_{kk} is equal to the susceptance of line k and non-diagonal elements are zero)
- $[A]$: M x n bus-branch **incidence matrix**

describe the connection of buses

Example

Same 4-bus network:



INCIDENCE MATRIX A

$$A = \begin{array}{c} \text{Bus:} \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \text{Br:} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}$$

SOLUTION $[P_{\text{br}}] = ([b] \times [A])[\theta]$

$$= \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -0.025 \\ -0.15 \\ -0.025 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 1.25 \\ 1.25 \\ 1.5 \end{bmatrix} \text{ pu}$$

Practical power flow diagram of power system